



\mathcal{I} -LACUNARY STATISTICAL CONVERGENCE OF WEIGHTED g VIA MODULUS FUNCTIONS IN 2-NORMED SPACES

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ABSTRACT. In this paper, we introduce new concepts of \mathcal{I} -statistical convergence and \mathcal{I} -lacunary statistical convergence using weighted density via modulus functions. Also, we study the relationship between them and obtain some interesting results.

1. INTRODUCTION AND PRELIMINARIES

Statistical convergence introduced by Fast [4] and Steinhaus [21] has many applications in different areas. Later on, this concept was reintroduced by Schoenberg in his own study [20]. The concept of statistical convergence is defined depending upon the natural density of the set $\Phi \subseteq \mathbb{N}$. The upper and lower natural density of the subset Φ is defined by

$$\bar{\delta}(\Phi) = \limsup_{n \rightarrow \infty} \frac{\Phi(1, n)}{n} \quad \text{and} \quad \underline{\delta}(\Phi) = \liminf_{n \rightarrow \infty} \frac{\Phi(1, n)}{n},$$

where $\Phi(1, n)$ denotes the number of elements in $\Phi \cap [1, n]$. If $\bar{\delta}(\Phi)$ and $\underline{\delta}(\Phi)$ are equal to each other, then the natural density of Φ exists and we denote it by $\delta(\Phi)$.

Obviously, $\delta(\Phi) = \lim_{n \rightarrow \infty} \frac{\Phi(1, n)}{n}$.

Utilizing above information, we say that a sequence $(x_k)_{k \in \mathbb{N}}$ is statistically convergent to x provided that for every $\varepsilon > 0$,

$$\delta(\{k \in \mathbb{N} : |x_k - x| \geq \varepsilon\}) = 0.$$

If $(x_k)_{k \in \mathbb{N}}$ is statistically convergent to x we write $st\text{-}\lim x_k = x$. For more detail informations about statistical convergent, see, in [9, 25, 26].

On the other hand, \mathcal{I} -convergence in a metric space was introduced by Kostyrko et al. [10] and its definition is depending upon the definition of an ideal \mathcal{I} in \mathbb{N} .

A family $\mathcal{I} \subset 2^{\mathbb{N}}$ is called an ideal if the following properties are held:

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- (i) $\emptyset \notin \mathcal{I}$;
- (ii) $P \cup R \in \mathcal{I}$ for every $P, R \in \mathcal{I}$;
- (iii) $R \in \mathcal{I}$ for every $P \in \mathcal{I}$ and $R \subset P$.

A non-empty family of sets $\mathcal{F} \subset 2^{\mathbb{N}}$ is a filter if and only if $\emptyset \notin \mathcal{F}$, $P \cap R \in \mathcal{F}$ for every $P, R \in \mathcal{F}$, and $R \in \mathcal{F}$ for every $P \in \mathcal{F}$ and every $R \supset P$. An ideal \mathcal{I} is said to be non-trivial if $\mathcal{I} \neq \emptyset$ and $X \notin \mathcal{I}$. The $\mathcal{I} \subset 2^X$ is a non-trivial ideal if and only if $\mathcal{F} = \mathcal{F}(\mathcal{I}) = \{\mathbb{N} \setminus P : P \in \mathcal{I}\}$ is a filter on X .

Definition 1. ([10]) *A sequence of reals $\{x_n\}_{n \in \mathbb{N}}$ is called the \mathcal{I} -convergent to L if, for each $\varepsilon > 0$, the set*

$$\Phi(\varepsilon) = \{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\} \in \mathcal{I}.$$

For more information about \mathcal{I} -convergent, see the references in [11, 12, 18].

The concept of the lacunary statistical convergence introduced in [5] is as follows: A lacunary sequence is an increasing sequence $\theta = \{k_r\}_{r \in \mathbb{N} \cup \{0\}}$ with property that $k_0 = 0$ and $h_r = k_r - k_{r-1} \rightarrow \infty$, as $r \rightarrow \infty$.

Let $I_r = (k_{r-1}, k_r]$ and $q_r = \frac{k_r}{k_{r-1}}$. A sequence $(x_k)_{k \in \mathbb{N}}$ is lacunary statistically convergent to x provided that for every $\varepsilon > 0$,

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} |\{k \in I_r : |x_k - x| \geq \varepsilon\}| = 0.$$

In [5], the authors discussed the relation between statistical convergence and lacunary statistical convergence. More results about lacunary statistical convergence can be found in [14, 15, 16, 22, 23].

Later on, the notions of \mathcal{I} -statistical convergence and \mathcal{I} -lacunary statistical convergence which extended the concepts of above mentioned convergence were given in [3, 17] and interesting results about these concepts were obtained.

In the recent times in [1], using the natural density of weight g where $g : \mathbb{N} \rightarrow [0, \infty)$ is a function with property that $\lim_{n \rightarrow \infty} g(n) = \infty$ and $\frac{n}{g(n)} \rightarrow 0$ as $n \rightarrow \infty$, the concept of natural density was extended as follows: The upper density of weight g was defined by

$$\bar{\delta}_g(\Phi) = \limsup_{n \rightarrow \infty} \frac{\Phi(1, n)}{g(n)}$$

for $\Phi \subset \mathbb{N}$, where $\Phi(1, n)$ denotes the number of elements in $\Phi \cap [1, n]$. The lower density of weight g is defined in a similar manner. Then, the family

$$\mathcal{I}_g = \{\Phi \subset \mathbb{N} : \bar{\delta}_g(\Phi) = 0\}$$

creates an ideal. It was seen in [1] that $\mathbb{N} \in \mathcal{I}_g$ iff $\frac{n}{g(n)} \rightarrow 0$ as $n \rightarrow \infty$. Furthermore, we suppose that $n/g(n) \rightarrow 0$ as $n \rightarrow \infty$ so that $\mathbb{N} \notin \mathcal{I}_g$ and \mathcal{I}_g is a proper admissible ideal of \mathbb{N} . We denote by G the collection of such weight functions g satisfying the above properties.

Very recently in [2], a new kind of density was defined by using the modulus function and the weighted function as follows: The modulus functions are defined as functions $f : \mathbb{R}^+ \cup \{0\} \rightarrow \mathbb{R}^+ \cup \{0\}$ satisfying the following properties.

- (i) f is increasing
- (ii) $f(x + y) \leq f(x) + f(y)$ for all $x, y \in \mathbb{R}^+$
- (iii) $f(x) = 0 \Leftrightarrow x = 0$
- (iv) f is right continuous at 0.

Then the $\bar{\delta}_g^f(\Phi)$ was defined in [2] by

$$\bar{\delta}_g^f(\Phi) = \lim_{n \rightarrow \infty} \frac{f(\Phi(1, n))}{f(g(n))}.$$

All the conditions to be a density function are satisfied by this new density function $\bar{\delta}_g^f(\Phi)$ apart from that $\frac{f(n)}{f(g(n))}$ as $n \rightarrow \infty$ might not be equal to 1. Also, the modulus function f applied to have the generalized density function as above must be unbounded. Otherwise, take $|f(x)| \leq M$ for all x and some $M > 0$. Then for any $\Phi \subseteq \mathbb{N}$, $\frac{f(\Phi(1, n))}{f(g(n))} \leq \frac{M}{f(g(n))} < \frac{M}{g(n)} \rightarrow 0$ which is of no interest again. Hence, we suppose that the modulus function to be unbounded. Moreover, the family

$$\mathcal{I}_g(f) = \left\{ \Phi \subset \mathbb{N} : \bar{\delta}_g^f(\Phi) = 0 \right\}$$

constructs an ideal. As a natural result, we can define the following definition.

Definition 2. A sequence (x_n) of real numbers is called the $\bar{\delta}_g^f$ -statistically convergent to x if for any $\varepsilon > 0$, $\bar{\delta}_g^f(\Phi(\varepsilon)) = 0$, where $\Phi(\varepsilon)$ is as in Definition 1.

Now we recall some basic definitions and notations.

The following 2-normed space was given by Gähler [6].

Definition 3. Let X be a real vector space of dimension d , where $2 \leq d < \infty$. A 2-norm on X is a function $\|\cdot, \cdot\| : X \times X \rightarrow \mathbb{R}$ which satisfies (i) $\|x, y\| = 0$ if and only if x and y are linearly dependent; (ii) $\|x, y\| = \|y, x\|$; (iii) $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in \mathbb{R}$; (iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$. The pair $(X, \|\cdot, \cdot\|)$ is then called a 2-normed space.

After this definition, many authors studied statistical convergence, \mathcal{I} -convergence, \mathcal{I} -Cauchy sequence, \mathcal{I}^* -convergent and \mathcal{I}^* -Cauchy sequence on this space (see [7, 8, 13]).

Definition 4. ([24]) Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal in \mathbb{N} . The sequence (x_k) of X is called the \mathcal{I} -statistically convergent to ξ , if for each $\varepsilon > 0$, $\delta > 0$ and nonzero z in X the set

$$\left\{ n \in \mathbb{N} : \frac{1}{n} |\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}| \geq \delta \right\} \in \mathcal{I}$$

or equivalently if for each $\varepsilon > 0$

$$\delta_{\mathcal{I}}(\Phi_n(\varepsilon)) = \mathcal{I}\text{-}\lim \delta_n(\Phi_n(\varepsilon)) = 0$$

where $\Phi_n(\varepsilon) = \{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}$ and $\delta_n(\Phi_n(\varepsilon)) = \frac{|\Phi_n(\varepsilon)|}{n}$.

If (x_k) is \mathcal{I} -statistically convergent to ξ then we write $\mathcal{I}\text{-st-}\lim_{k \rightarrow \infty} \|x_k - \xi, z\| = 0$ or $\mathcal{I}\text{-st-}\lim_{k \rightarrow \infty} \|x_k, z\| = \|\xi, z\|$.

In this paper, we introduce new concepts of \mathcal{I} -statistical convergence and \mathcal{I} -lacunary statistical convergence using weighted density via modulus functions. Also, we study the relationship between them and obtain some interesting results.

2. MAIN RESULTS

Our main definitions and notations are as following:

Definition 5. Let θ be a lacunary sequence. A sequence $\{x_k\}$ in 2-normed space $(X, \|\cdot, \cdot\|)$ is said to be \mathcal{I} -lacunary statistically convergent to ξ if for every $\varepsilon > 0$, $\delta > 0$ and every nonzero $z \in X$,

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r} |\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}| \geq \delta \right\}$$

belongs to \mathcal{I} . In this case, we write $x_k \rightarrow x(S_{\theta}(\mathcal{I}))$.

Definition 6. A sequence $\{x_k\}$ in 2-normed space $(X, \|\cdot, \cdot\|)$ is called as \mathcal{I} -statistically convergent of weight g via modulus function f to ξ if for every $\varepsilon > 0$, $\delta > 0$ and every nonzero $z \in X$,

$$\left\{ n \in \mathbb{N} : \frac{f(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(n))} \geq \delta \right\}$$

belongs to \mathcal{I} . In this case, we write $x_k \rightarrow x(S(\mathcal{I}_g(f)))$.

The $S(\mathcal{I}_g(f))$ will denote the set of all \mathcal{I} -statistically convergent sequences of weight g via modulus function f .

Remark 7. For $\mathcal{I} = \mathcal{I}_{fin} = \{\Phi \subseteq \mathbb{N} : \Phi \text{ is finite set}\}$, $S(\mathcal{I}_g(f))$ -convergence coincides with statistical convergence of weight g via modulus function f which has so far not been studied. Also taking $f_{\alpha}(x) = x^{\alpha}$ for $\alpha \in (0, 1)$, it reduces to \mathcal{I} -statistically convergent of weight g .

Definition 8. Let θ be a lacunary sequence. A sequence $\{x_k\}$ in 2-normed space $(X, \|\cdot, \cdot\|)$ is called as \mathcal{I} -lacunary statistically convergent of weight g via modulus function f to ξ if for every $\varepsilon > 0$, $\delta > 0$ and every nonzero $z \in X$,

$$\left\{ r \in \mathbb{N} : \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} \geq \delta \right\}$$

belongs to \mathcal{I} . In this case, we write $x_k \rightarrow x(S_{\theta}(\mathcal{I}_g(f)))$.

The $S_\theta(\mathcal{I}_g(f))$ will denote the set of all \mathcal{I} -lacunary statistically convergent sequences of weight g via modulus function f .

Now we are ready to give main results.

Theorem 9. *Let f_1 and f_2 are two modulus functions and let $g_1, g_2 \in G$ be such that there are $\lambda_1, \lambda_2 > 0, j_0 \in \mathbb{N}$ for which $\frac{f_1(x)}{f_2(x)} \geq \lambda_1$ for every x and $\frac{f_1(g_1(n))}{f_2(g_2(n))} \leq \lambda_2$ for every $n \geq j_0$. Then $S(\mathcal{I}_{g_1}(f_1)) \subset S(\mathcal{I}_{g_2}(f_2))$.*

Proof. For any $\varepsilon > 0$ and every $z \in X$,

$$\begin{aligned} & \frac{f_2(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f_2(g_2(n))} \\ &= \frac{f_2(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f_1(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)} \cdot \frac{f_1(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f_1(g_1(n))} \cdot \frac{f_1(g_1(n))}{f_2(g_2(n))} \\ &\leq \frac{\lambda_2 f_1(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{\lambda_1 f_1(g_1(n))} \end{aligned}$$

for $n \geq j_0$. Therefore for any $\delta > 0$

$$\begin{aligned} & \left\{ n \in \mathbb{N} : \frac{f_2(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f_2(g_2(n))} \geq \delta \right\} \\ & \subset \left\{ n \in \mathbb{N} : \frac{f_1(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f_1(g_1(n))} \geq \delta \frac{\lambda_1}{\lambda_2} \right\} \cup \{1, 2, \dots, j_0\}. \end{aligned}$$

If $(x_k) \in S(\mathcal{I}_{g_1}(f_1))$, then the set on the right side belongs to \mathcal{I} and hence the set on the left side belongs to \mathcal{I} which gives us that $S(\mathcal{I}_{g_1}(f_1)) \subset S(\mathcal{I}_{g_2}(f_2))$. \square

Definition 10. *Let θ be a lacunary sequence. A sequence $\{x_k\}$ in 2-normed space $(X, \|\cdot, \cdot\|)$ is said to be strongly \mathcal{I} -lacunary convergent of weight g via modulus function f to ξ if for every $\varepsilon > 0$ and every nonzero $z \in X$,*

$$\left\{ r \in \mathbb{N} : \frac{1}{f(g(h_r))} \sum_{k \in I_r} f(\|x_k - \xi, z\|) \geq \varepsilon \right\}$$

belongs to \mathcal{I} . In this case, we write $x_k \rightarrow x(N_\theta(\mathcal{I}_g(f)))$ and the set of such sequences will be represented by $N_\theta(\mathcal{I}_g(f))$.

Theorem 11. *Let θ be a lacunary sequence, and $\{x_k\}$ in 2-normed space $(X, \|\cdot, \cdot\|)$ be a sequence. Then $x_k \rightarrow x(N_\theta(\mathcal{I}_g(f)))$ means that $x_k \rightarrow x(S_\theta(\mathcal{I}_g(f)))$.*

Proof. Let $x_k \rightarrow x(N_\theta(\mathcal{I}_g(f)))$. Then we have for any $\varepsilon > 0$ and every $z \in X$

$$\begin{aligned} \sum_{k \in I_r} f(\|x_k - \xi, z\|) &\geq \sum_{k \in I_r, f(\|x_k - \xi, z\|) \geq \varepsilon} f(\|x_k - \xi, z\|) \\ &\geq \varepsilon |\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}| \end{aligned}$$

and hence

$$\frac{1}{\varepsilon \cdot f(g(h_r))} \sum_{k \in I_r} f(\|x_k - \xi, z\|) \geq \frac{1}{f(g(h_r))} f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|).$$

Therefore, we get for any $\delta > 0$ and every $z \in X$

$$\begin{aligned} & \left\{ r \in \mathbb{N} : \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} \geq \delta \right\} \\ & \subseteq \left\{ r \in \mathbb{N} : \frac{1}{f(g(h_r))} \sum_{k \in I_r} f(\|x_k - \xi, z\|) \geq \delta \cdot \varepsilon \right\} \in \mathcal{I} \end{aligned}$$

which gives the desired result. □

Now we give the relationship between \mathcal{I} -statistical and \mathcal{I} -lacunary statistical convergence of weight g via modulus function f .

Theorem 12. *Let $(X, \|\cdot, \cdot\|)$ be a 2-normed space and let θ be a lacunary sequence. Then $x_k \rightarrow x (S(\mathcal{I}_g(f)))$ means that $x_k \rightarrow x (S_\theta(\mathcal{I}_g(f)))$ provided that*

$$\liminf_r \frac{f(g(h_r))}{f(g(k_r))} > 1.$$

Proof. Because of the condition, we can find a $M > 1$ such that for sufficiently large r we get $\frac{f(g(h_r))}{f(g(k_r))} \geq M$.

Because of $x_k \rightarrow x (S(\mathcal{I}_g(f)))$, we get for every $\varepsilon > 0$ and sufficiently large r

$$\begin{aligned} \frac{1}{f(g(k_r))} f(|\{k \leq k_r : \|x_k - \xi, z\| \geq \varepsilon\}|) & \geq \frac{1}{f(g(k_r))} f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|) \\ & \geq M \frac{1}{f(g(h_r))} f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|). \end{aligned}$$

Then we obtain that for any $\delta > 0$

$$\begin{aligned} & \left\{ r \in \mathbb{N} : \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} \geq \delta \right\} \\ & \subseteq \left\{ r \in \mathbb{N} : \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(k_r))} \geq M\delta \right\} \end{aligned}$$

belongs to \mathcal{I} , which gives $x_k \rightarrow x (S_\theta(\mathcal{I}_g(f)))$, as desired. □

For the following result, suppose that θ be a lacunary sequence such that for any $A \in \mathcal{F}(\mathcal{I})$

$$\bigcup \{n : k_{r-1} < n < k_r, r \in A\} \in \mathcal{F}(\mathcal{I})$$

as in [19].

Theorem 13. Let $(X, \|\cdot, \cdot\|)$ be a 2-normed space and let θ be a lacunary sequence. Then $x_k \rightarrow x (S_\theta(\mathcal{I}_g(f)))$ means that $x_k \rightarrow x (S(\mathcal{I}_g(f)))$ provided that

$$\sup_r \sum_{i=0}^{r-1} \frac{f(g(h_{i+1}))}{f(g(k_{r-1}))} = B < \infty.$$

Proof. Assume that $x_k \rightarrow x (S_\theta(\mathcal{I}_g(f)))$ and $\varepsilon, \delta, \delta_1 > 0$ define the sets

$$A = \left\{ r \in \mathbb{N} : \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} < \delta \right\}$$

and

$$B = \left\{ n \in \mathbb{N} : \frac{f(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} < \delta_1 \right\}.$$

It is clear that $A \in \mathcal{F}(\mathcal{I})$ from our assumption. Furthermore,

$$T_i = \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} < \delta$$

for all $j \in A$. Let $n \in \mathbb{N}$ be such that $k_{r-1} < n < k_r$ for some $r \in A$. Then

$$\begin{aligned} & \frac{f(|\{k \leq n : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(n))} \\ & \leq \frac{f(|\{k \leq k_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(k_{r-1}))} \\ & = \frac{f(|\{k \in I_1 : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(k_{r-1}))} + \dots + \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(k_{r-1}))} \\ & = \frac{f(g(k_1))}{f(g(k_{r-1}))} \frac{f(|\{k \in I_1 : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_1))} \\ & + \frac{f(g(k_2 - k_1))}{f(g(k_{r-1}))} \frac{f(|\{k \in I_2 : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_2))} \\ & + \dots + \frac{f(g(k_r - k_{r-1}))}{f(g(k_{r-1}))} \frac{f(|\{k \in I_r : \|x_k - \xi, z\| \geq \varepsilon\}|)}{f(g(h_r))} \\ & = \frac{f(g(k_1))}{f(g(k_{r-1}))} T_1 + \frac{f(g(k_2 - k_1))}{f(g(k_{r-1}))} T_2 + \dots + \frac{f(g(k_r - k_{r-1}))}{f(g(k_{r-1}))} T_r \\ & \leq \sup_{j \in A} T_j \cdot \sup_r \sum_{i=0}^{r-1} \frac{f(g(h_{i+1}))}{f(g(k_{r-1}))} < B\delta. \end{aligned}$$

Taking $\delta_1 = \frac{\delta}{B}$ and considering $\bigcup \{n : k_{r-1} < n < k_r, r \in A\} \subset B$ where $\mathcal{F}(\mathcal{I})$ it follows from assumption the set B also belongs to $\mathcal{F}(\mathcal{I})$ which finishes the proof. \square

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