Value of the $\xi$ Parameter in $K_{13}$ Decay from the Pion-Gauge Condition*

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It is shown with the help of the pion-gauge condition that the divergence of the most general boson current in $K_{13}$ decay vanishes in the limit of zero four-momentum of the pion. This in turn implies $\xi = -1$ at the (unphysical) point $t = m^2$. Inasmuch as $\xi$ does not seem, experimentally, to vary much with $t$, the result is significant in the physical region and agrees with experiment.

Recently, Fischbach et al.\textsuperscript{1,2} reported reasonably good values for some of the $K_{13}$ decay parameters by assuming a zero in the so-called scalar form factor $f_s(t) = (m_b^2 - m^2)f_k(t)$ at the threshold $t = t_0 = (m_b + m)^2$, which they obtained using part of the Kemmer currents for bosons. It was then correctly pointed out\textsuperscript{3} that the vanishing of $f_s(t)$ at $t = t_0$ cannot be purely kinematical, as claimed.

We show in this note that the most general boson current in Kemmer form coupled to leptons is conserved, not at $t = t_0$ as previously claimed,\textsuperscript{1,2} but at the (unphysical) point $p^0_f = 0$. This is achieved by applying the so-called "pion-gauge condition" to the $K_{13}$ decay. According to this condition the invariant amplitude vanishes at the (unphysical) point $p^0_f = 0$.\textsuperscript{4}

If we impose this condition, we find a zero in $f_s(t)$ at $t = t_0^* = m^2$, i.e., the zero-pion-mass limit of $t_0 = (m_b + m)^2$ of Ref. 1. This condition gives us, as we shall see, a value $\xi(m_b^2) = f_s/t_s = -1$, instead of $\xi(m_b^2)/f_s = -1$ obtained at the (unphysical) point $t = t_0$ of Ref. 1 (which by the way goes over to our value for $m_b^2 = 0$). Inasmuch as $\xi$ does not seem to depend much experimentally on $t$ (see Fig. 1), our result is also significant in the physical region.

We now show the details of the argument. There are three vector currents in the Kemmer formalism:

\begin{align}
J^\lambda_1 &= i\bar{u}_s\gamma^\lambda u_K = \frac{i}{2}(p_K^\mu/m_K + p_s^\mu/m_s), \\
J^\mu_2 &= q/2\bar{u}_s m_K/((m_K + m_s)m_s/((m_K + m_s)), \\
J^\lambda_3 &= i\bar{u}_s\gamma^\lambda C u_K = \frac{i}{2}(p_K^\mu/m_K - p_s^\mu/m_s),
\end{align}

where $q = p_K - p_s$, $t = m^2$, and $C$ is a $5 \times 5$ matrix given explicitly in Ref. 9. Fischbach et al.\textsuperscript{1,2} have chosen only the first two terms. However, the last term can be written as

\begin{equation}
J^\lambda_3 = -m_K - m_s \frac{m_K - m_s}{m_K + m_s} J^\mu_2 + \frac{4m_K m_s}{l_0 - l} J^\mu_2.
\end{equation}

Now if one starts with the combination

\begin{equation}
V^\lambda = g_3(t) J^\lambda_1 + g_2(t) J^\mu_2 + g_3(t) J^\mu_3,
\end{equation}

which can be rearranged as

\begin{align}
V^\lambda &= \left[ g_3(t) - g_2(t) \frac{m_K - m_s}{m_K + m_s} \right] J^\lambda_1 \\
&+ \left[ g_2(t) - g_3(t) \frac{4m_K m_s}{l_0 - l} \right] J^\mu_2,
\end{align}

or

\begin{align}
V^\lambda &= g_3(t) J^\lambda_1 + g_2(t) J^\mu_2, \\
&+ \left[ g_2(t) - g_3(t) \frac{4m_K m_s}{l_0 - l} \right] \times \frac{m_K - m_s}{m_K + m_s} J^\mu_2,
\end{align}

one concludes\textsuperscript{3} that the basic assumption of Fischbach et al.\textsuperscript{1,2} that $g_3(t)$ should be regular at $t = t_0$ fails if $g_3(t)$ has no zero at $t = t_0$, or if $g_3(t)$ does not have a pole at $t = t_0$ with the exact residue $4m_K m_s g_3(t_0)$. (This conclusion is also true for vanishing $g_2$ or $g_3$ or both.) In Eq. (6), the connection between $f_s$ and $g_3$ is given by

\begin{align}
(4m_K m_s)^{1/2} f_s &= (m_K + m_s)g_3(t), \\
(4m_K m_s)^{1/2} (m_K + m_s) f_s &= (m_K^2 - m_s^2)g_3(t) - (l - l_0)g_3(t).
\end{align}

Now, the so-called scalar form factor $f_s(t)$ is the divergence of the matrix element of the boson current in $K_{13}$ decay. One readily verifies that

\begin{align}
\partial_t J^\lambda_1 &= \frac{m_K - m_s}{4m_K m_s} (l_0 - l), \\
\partial_t J^\mu_3 &= \frac{l(t_0 - l)}{4m_K m_s (m_K + m_s)}, \\
\partial_t J^\lambda_3 &= \frac{m_K + m_s}{4m_K m_s} \left[ (l - (m_K - m_s)^2 \right].
\end{align}

It then follows from (7), (8), and (6) that if one takes the $(K, n)$ current as a linear combination of
\[ f_+(t = m_K^2) + f_-(t = m_K^2) = 0 \]  
(13)

or

\[ \xi(m_K^2) = f_-/f_+ \big| t = m_K^2 = -1. \]  
(13')

By substituting the result (13) into the so-called scalar form factor \( f_0(t) \) we find that \( f_0(t) \) vanishes in the limit \( \rho_\pi^2 \to 0 \). Thus, because \( f_0(t) \) is proportional to the divergence of the matrix element of the boson current in \( K_{13} \) decay, the boson current coupled to the lepton current is conserved at the (unphysical) point \( \rho_\pi^2 \to 0 \), i.e.,

\[ \lim_{\rho_\pi^2 \to 0} g_i(t) \delta^3 \mathcal{J}_1 = 0, \quad i = 1, 2, 3. \]  
(14)

Equations (4)–(9), when compared with (14), yield the restrictions on the \( g_i(t) \) imposed by the pion-gauge condition.

We should like to remark that the above considerations are essentially \( S \)-matrix considerations, the currents being used to write down the most general vertex. If, however, we postulate a field theory with an interaction Lagrangian using one of the currents in Eqs. (1)–(3), then, to lowest order, we can make predictions on the value of \( \xi \), the form factors being constants in the lowest order:

<table>
<thead>
<tr>
<th>Interaction current</th>
<th>( \xi )</th>
</tr>
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<tbody>
<tr>
<td>( J_1 )</td>
<td>-0.57</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>( \infty ) (( f_0 \neq 0 ))</td>
</tr>
<tr>
<td>( J_3 )</td>
<td>-1.75</td>
</tr>
</tbody>
</table>

An example of a perturbation theory of weak interactions based on \( J_3 \) alone was given some time ago in Ref. 9. The calculations with \( J_3 \) alone are reported elsewhere.\(^10\)

The pion-gauge condition, variously derived or justified from different points of view,\(^5\)–\(^7\) and applied to several other processes,\(^8\) is not generally used. It is, of course, different from and stronger than the usual soft-pion limit of current algebra. This is why the result (13) or (13') is different from the Callan-Treiman-Suzuki-Mathur-Okubo-Pandit relation.\(^11\) The apparent agreement of the result (13') with experiment\(^12\) (Fig. 1) may be considered as a support for the validity of the pion-gauge condition.

Finally, we mention that some time ago Kang\(^13\) postulated phenomenologically a zero in \( f_0(t) \) between \( (m_K - m_\pi)^2 \) and \( (m_K + m_\pi)^2 \) in order to explain the experimental value \( \xi(0) \approx -1 \). The pion-gauge condition provides a theoretical basis for this assumption. Kang shows furthermore that the theoretical arguments based on the soft-pion limit of the SU(3)×SU(3) current algebra is not sufficient to conclude \( \xi = -1 \), i.e., the Callan-Treiman-
Unified Explanation of the $K_L^0 \rightarrow 2\mu$ Puzzle, CP Nonconservation, and the Excess Muon Anomaly

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We propose a common explanation of (a) the $K_L^0 \rightarrow \mu^+\mu^-$ puzzle, (b) CP nonconservation, and (c) the cosmic-ray muon anomaly of Utah. The model involves a pair of neutral bosons with pairwise strong interactions with known hadrons. Experimental implications are discussed.

It is the purpose of the present note to study the consequences of assuming that the following three outstanding puzzles have a unified explanation. These puzzles are

(a) the anomalously low branching ratio of $K_L^0 \rightarrow \mu^+\mu^-$; viz., B. R. ($K_L^0 \rightarrow \mu^+\mu^-) < 1.8 \times 10^{-9}$ (90% confidence),
(b) the long-standing search for the mechanism leading to CP nonconservation in $K_L^0$ decays,
(c) the Utah effect or the unexpected isotropic component in cosmic-ray muons for muon energies $\approx 1$ TeV.

That (a) and (b) may be related was first stressed by Christ and Lee. A specific model in terms of CP-violating neutral currents which synthesizes (a) and (b) has been suggested by Wolfenstein recently. The proposition that (c) the $X$ particle responsible for the Utah effect is also involved in resolution of (a) and (b) is of course a purely theoretical speculation. It is however consistent with existing experiments and can be tested experimentally.

THE MODEL

Wolfenstein pointed out that the most economic way to understand the $K_L^0 \rightarrow 2\mu$ puzzle in terms of CP nonconservation [(a) and (b)] is to postulate that there exists a $CP$-noninvariant interaction $H'$ involving a neutral muon current which allows the decay $K_L^0 \rightarrow \mu^+\mu^-$ but not the decay $K_L^0 \rightarrow \mu^+\mu^-$. A simple model for $H'$ satisfying these constraints is

$$H' = G' \sin \theta \bar{A}_L \bar{A}_R \gamma_\lambda \gamma_5 \psi_\mu \bar{\psi}_\mu,$$

$$A_L^2 = (A_L^2 + A_R^2) / \sqrt{2},$$

where $A_L$ is the seventh component of the hadronic