# A search for the fourth SM family 

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#### Abstract

Arguments favouring the existence of a fourth fermion family in the framework of the Standard Model are given. The masses of the fourth family neutrino, charged lepton and down quark are close to each other within an accuracy of the order of a few GeV and lie between $300-700 \mathrm{GeV}$. Experiments on the search for these particles are discussed.


The mass spectrum and the mixings of fundamental fermions seem to be the most important unsolved problem of particle physics. According to the Standard Model (SM), these masses and mixings arise from the interaction with the Higgs doublet via spontaneous symmetry breaking. In general, a large number of parameters is put in by hand in order to explain reality.

On the other hand, before the symmetry breaking, fermions with the same quantum numbers (electric charge, weak isospin, etc.) are indistinguishable. Therefore, in the fermion-Higgs interaction the Lagrangian terms corresponding to fermions with the same quantum numbers should come with equal strength. Consequently, after the spontaneous symmetry breaking one deals with singular mass matrices in which all entries are equal to $a \eta$, where $a$ is the strength of the fermion-Higgs interaction and $\eta$ is the vacuum expectation value of the Higgs field. It seems natural to take $a$ equal to the $\operatorname{SU}(2)$ gauge coupling constant $g$. According to this approach, in the case of $n$ SM families, $n-1$ families are massless and the $n$th family fermions have masses $n g \eta$. Taking the real mass spectrum of the third family fermions into account necessarily leads to the assumption that at least a fourth SM family must exist.

Similar arguments on the mass spectrum of the fundamental fermions have been suggested by high energy physicists during the last fifteen years [1-5]. In general, the authors tend to apply the democratic mass matrix (DMM) approach to the first three families only. As a consequence, the extension of the SM becomes unavoidable and/or wrong results have been obtained. For example, in Refs. [1-4] the $S U_{L}(2) \times S U_{R}(2) \times U(1)$ extension of the SM gauge group was considered, in Refs. [1,2] the mass of the $t$ quark was predicted to be $11-14 \mathrm{GeV}$, etc.
For the four family SM considered here, the DMM approach leads to

$$
M^{0}=g \eta\left(\begin{array}{llll}
1 & 1 & 1 & 1  \tag{1}\\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1
\end{array}\right) \rightarrow M=4 g \eta\left(\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

[^0]for all types of fundamental fermions. As can be seen, the first three families remain massless and the fourth family fermions accept the masses $4 g \eta=8 m_{W} \simeq 640 \mathrm{GeV}$. These values are close to the critical fermion mass values established using partial-wave unitarity at high energy [6]. It should be noted that the coupling constant in Eq. (1), in principle, can differ from $g$. If $g$ is replaced by the electromagnetic coupling constant $e$, then $m_{4} \simeq 320 \mathrm{GeV}$. In general, we expect that the fourth family fermion masses lie between 300 and 700 GeV .
It is clear that Eq. (1) reflects reality with high accuracy for the neutrinos. The accuracy is quite sufficient also for the charged leptons and down quarks. But, in the up quark sector this attractive picture is broken by the $t$ quark mass $m_{t}=175 \mp 25 \mathrm{GeV}$ [7]. The possible sources of this breaking will be considered elsewhere.

The transformation in Eq. (1) is performed by using a $4 \times 4$ orthogonal matrix. In general, the $\mathrm{O}(4)$ rotations can be expressed in terms of six angles. Among the available parameterizations, the most appropriate one for our purpose is the following:

$$
\begin{equation*}
O=R_{12}^{\mathrm{T}} R_{13}^{\mathrm{T}} R_{23}^{\mathrm{T}} R_{14}^{\mathrm{T}} R_{24}^{\mathrm{T}} R_{34}^{\mathrm{T}}, \tag{2}
\end{equation*}
$$

where $R_{i j}$ denotes a rotation in the ( $i-j$ ) plane. For example

$$
R_{34}=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & c_{34} & s_{34} \\
0 & 0 & -s_{34} & c_{34}
\end{array}\right)
$$

Here $c_{i j}=\cos \theta_{i j}$ and $s_{i j}=\sin \theta_{i j}$. By grouping the first and the last three rotations in Eq. (2), we obtain

$$
\begin{align*}
O= & O_{a} O_{b}=\left(\begin{array}{cccc}
c_{12} c_{13} & -s_{12} c_{23}-s_{13} s_{23} c_{12} & s_{12} s_{23}-s_{13} c_{12} c_{23} & 0 \\
s_{12} s_{13} & -s_{12} s_{23} s_{13}+c_{12} c_{23} & -s_{12} s_{13} c_{23}-s_{23} c_{12} & 0 \\
s_{13} & s_{23} c_{13} & c_{13} c_{23} & 0 \\
0 & 0 & 0 & 1
\end{array}\right) \\
& \times\left(\begin{array}{ccc}
c_{14} & -s_{14} s_{24} & -s_{14} s_{34} c_{24} \\
0 & c_{24} & -s_{14} c_{24} c_{34} \\
0 & 0 & s_{24} s_{34} \\
s_{14} & s_{24} c_{14} & -s_{24} c_{24} c_{14} c_{24} \\
s_{34} & c_{14} c_{34} c_{24}
\end{array}\right) . \tag{3}
\end{align*}
$$

At this stage we neglect the masses of the first three family neutrinos, charged leptons and down quarks. Indeed the masses of the heaviest first three family fermions $m_{\tau}=1.8 \mathrm{GeV}$ and $m_{b}=5 \mathrm{GeV}$ are much smaller than our natural fermion mass scale $g \eta=160 \mathrm{GeV}$. Therefore the mixings within the first three families shown as the first matrix $O_{a}$ in Eq. (3) remain arbitrary. On the other hand, Eq. (3) gives

$$
\begin{equation*}
f_{4}=s_{14} f_{1}^{0}+s_{24} c_{14} f_{2}^{0}+s_{34} c_{24} c_{14} f_{3}^{0}+c_{34} c_{24} c_{14} f_{4}^{0}, \tag{4}
\end{equation*}
$$

where $f_{4}$ denotes the mass eigenstate corresponding to $m_{4}=4 g \eta$ and the superscript zero denotes weak-eigenstates which are included in the initial Lagrangian. Comparing Eq. (4) with the eigenstate $f_{4}$ obtained directly from Eq. (1), we get

$$
\begin{equation*}
s_{14}=s_{24} c_{14}=s_{34} c_{24} c_{14}=c_{34} c_{24} c_{14}=\frac{1}{2} . \tag{5}
\end{equation*}
$$

As a result, the matrix $O_{b}$ in Eq. (3) becomes

$$
O_{b}=\left(\begin{array}{cccc}
\sqrt{3} / 2 & -\sqrt{3} / 6 & -\sqrt{3} / 6 & -\sqrt{3} / 6  \tag{6}\\
0 & \sqrt{6} / 3 & -\sqrt{6} / 6 & -\sqrt{6} / 6 \\
0 & 0 & \sqrt{2} / 2 & -\sqrt{2} / 2 \\
1 / 2 & 1 / 2 & 1 / 2 & 1 / 2
\end{array}\right) .
$$

Then, one can easily obtain the leptonic CKM matrix

$$
U^{\ell}=O_{\nu} O_{l}^{\mathrm{T}}=\left(\begin{array}{cccc}
c_{12}^{\prime} c_{13}^{\prime} & s_{12}^{\prime} s_{13}^{\prime} & s_{13}^{\prime} & 0  \tag{7}\\
-s_{12}^{\prime} c_{23}^{\prime}-s_{13}^{\prime} s_{23}^{\prime} c_{12}^{\prime} & -s_{12}^{\prime} s_{23}^{\prime} s_{13}^{\prime}+c_{12}^{\prime} c_{23}^{\prime} & s_{23}^{\prime} c_{13}^{\prime} & 0 \\
s_{12}^{\prime} s_{23}^{\prime}-s_{13}^{\prime} c_{12}^{\prime} c_{23}^{\prime} & -s_{12}^{\prime} s_{13}^{\prime} c_{23}^{\prime}-s_{23}^{\prime} c_{12}^{\prime} & c_{13}^{\prime} c_{23}^{\prime} & 0 \\
0 & 0 & 0 & 1
\end{array}\right) \text {, }
$$

where $\theta_{i j}^{\prime}=\theta_{i j}^{y}-\theta_{i j}^{l}$. As can be seen the fourth family is decoupled. This is a consequence of the negligence of the first three leptons' masses. In the DMM approach, we expect that the mixings of the fourth family with the first three should be given by $m_{e} / m_{4}, m_{\mu} / m_{4}$ and $m_{\tau} / m_{4}$, respectively.

Let us turn to the up quark sector. At this stage, we suppose that a non-zero $t$-quark mass appears due to the modification of the $\left(t^{0}, u_{4}^{0}\right)$ sector of the DMM. In other words, the up quark mass matrix has the following form:

$$
\left(\begin{array}{cccc}
1 & 1 & 1 & 1  \tag{8}\\
1 & 1 & 1 & 1 \\
1 & 1 & 1+\epsilon_{1} & 1+\epsilon_{2} \\
1 & 1 & 1+\epsilon_{3} & 1+\epsilon_{4}
\end{array}\right) .
$$

Since the mass of the $u$ and $c$ quarks are much smaller than 160 GeV , we can neglect them and this leads us to the condition $\epsilon_{1} \epsilon_{4}=\epsilon_{2} \epsilon_{3}$.

It can be easily seen that $\epsilon_{1}=\epsilon_{4}=-\epsilon_{2}=-\epsilon_{3}=\epsilon$ is one of the choices which satisfy the above condition. We prefer this choice because it effects minimally the DMM. In this case two mass eigenstates are different from zero: $m_{t}=2 \epsilon g \eta$ and $m_{\mu 4}=4 g \eta$. Taking the $t$ quark mass as $175 \mp 25 \mathrm{GeV}$ and $g \eta=2 m_{W} \simeq 160 \mathrm{GeV}$ gives $\epsilon \simeq 0.55 \mp 0.08$. With this choice, Eq. (6) remains unchanged and in Eq. (3) the angles $s_{12}$ and $s_{13}$ become zero. The CKM matrix of the quark sector has the same form as Eq. (7), with $\theta_{12}^{\prime}=\theta_{12}^{u}-\theta_{12}^{d}, \theta_{13}^{\prime}=-\theta_{13}^{d}$ and $\theta_{23}^{\prime}=-\theta_{23}^{d}$. With the chosen values of $\epsilon_{i}$, the fourth quark family is also decoupled like in the lepton case.

The fourth family fermions may manifest themselves as deviations of $\rho=m_{W}^{2} / m_{Z}^{2} \cos ^{2} \theta_{W}=1$. The heavy fermion doublet leads to the following correction to the parameter $\rho$ [6]:

$$
\begin{equation*}
\rho=1+\xi \frac{G_{F}}{8 \sqrt{2} \pi^{2}}\left[\frac{2 m_{1}^{2} m_{2}^{2}}{m_{1}^{2}-m_{2}^{2}} \ln \left(\frac{m_{2}^{2}}{m_{1}^{2}}\right)+m_{1}^{2}+m_{2}^{2}\right], \tag{9}
\end{equation*}
$$

where $\xi=1$ for leptons and $\xi=3$ for quarks, $m_{1}$ and $m_{2}$ are the masses of the up and down type heavy fermions in the doublet respectively. As can be seen from Eq. (9), for $m_{1}=m_{2}$ the correction is zero, for $m_{1}^{2} \ll m_{2}^{2}$ ( $m_{2}^{2} \ll$ $m_{1}^{2}$ )

$$
\rho=1+\xi \frac{G_{F}}{8 \sqrt{2} \pi^{2}} m_{2(1)}^{2} .
$$

Therefore, considerable contributions to $\rho$ will come from the $t$-quark only.
Production of the fourth family fermions. It is clear that direct pair production of fourth family fermions will be possible at future TeV energy colliders only. Linear $e^{+} e^{-}$colliders with $\sqrt{s} \geqslant 1.5 \mathrm{TeV}$ and sufficiently high luminosity will give the opportunity to search for all fermions from the fourth family. The cross-section for the process $e^{+} e^{-} \rightarrow f \bar{f}$ has the form [8]

$$
\sigma=\frac{2 \pi \alpha^{2}}{3 s} \xi \beta\left(Q_{f}\left(Q_{f}-2 \chi_{1} v v_{f}\right)\left(3-\beta^{2}\right)+\chi_{2}\left(1+v^{2}\right)\left[v_{f}^{2}\left(3-\beta^{2}\right)+2 \beta^{2} a_{f}^{2}\right]\right\},
$$

where

$$
\begin{aligned}
& \chi_{1}=\frac{1}{16 \sin ^{2} \theta_{W} \cos ^{2} \theta_{W}} \frac{s\left(s-M_{Z}^{2}\right)}{\left(s-M_{Z}^{2}\right)^{2}+\Gamma_{Z}^{2} M_{Z}^{2}}, \\
& \chi_{2}=\frac{1}{256 \sin ^{4} \theta_{W} \cos ^{4} \theta_{W}} \frac{s^{2}}{\left(s-M_{Z}^{2}\right)^{2}+\Gamma_{Z}^{2} M_{Z}^{2}}, \\
& v=-1+4 \sin ^{2} \theta_{W}, \quad a_{f}=2 T_{3 f}, \quad v_{f}=2 T_{3 f}-4 Q_{f} \sin ^{2} \theta_{W}, \quad \beta=\sqrt{1-4 m^{2} / s} .
\end{aligned}
$$

$T_{3}=\frac{1}{2}$ for $\nu_{4}$ and $u_{4}, T_{3}=-\frac{1}{2}$ for $\ell_{4}$ and $d_{4}$. For $\sqrt{s_{e e}}=2 \mathrm{TeV}, M_{Z}=91 \mathrm{GeV}, \sin ^{2} \theta_{W}=0.23$ and fourth family fermion masses of 640 GeV , we obtain $\sigma\left(\nu_{4} \bar{\nu}_{4}\right)=4 \mathrm{fb}, \sigma\left(\ell_{4}^{+} \ell_{4}^{-}\right)=20 \mathrm{fb}, \sigma\left(u_{4} \bar{u}_{4}\right)=30 \mathrm{fb}$ and $\sigma\left(d_{4} \bar{d}_{4}\right)=15 \mathrm{fb}$. With an $e^{+} e^{-}$luminosity of $10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$, these cross sections correspond to the production of $40 \nu_{4} \bar{\nu}_{4}, 200$ $\ell_{4}^{+} \ell_{4}^{-}, 300 u_{4} \bar{u}_{4}$ and $150 d_{4} \bar{d}_{4}$ per year, respectively.
It is well known that linear $e^{+} e^{-}$colliders will allow to construct TeV energy $\gamma \gamma$ colliders on their basis (see [9] and references therein). The achievable luminosities of $\gamma \gamma$ colliders are expected to be much higher than the luminosities of $e^{+} e^{-}$colliders. Therefore $\ell_{4}, d_{4}$ and $u_{4}$ will be produced in larger numbers at $\gamma \gamma$ machines. The cross-section for $\gamma \gamma \rightarrow f \bar{f}$ at fixed $\hat{s}$ has the form

$$
\begin{equation*}
\hat{\sigma}=\frac{2 \xi \pi \alpha_{\mathrm{em}}^{2} Q^{4}}{\hat{s}\left(1+\beta^{2}\right)}\left[2 \beta\left(\beta^{4}-\beta^{2}-2\right)+\left(\beta^{6}+\beta^{4}-3 \beta^{2}-3\right) \ln \left(\frac{1-\beta}{1+\beta}\right)\right], \tag{10}
\end{equation*}
$$

where $\beta=\sqrt{1-4 m^{2} / s}$. Further integration over the photon spectrum should be performed to obtain the resulting cross section

$$
\begin{equation*}
\sigma=\int_{\tau_{\min }}^{(0.83)^{2}} \mathrm{~d} \tau \int_{\tau / 0.83}^{0.83} \frac{\mathrm{~d} x}{x} f_{\gamma}\left(\frac{\tau}{x}\right) f_{\gamma}(x) \hat{\sigma}(\tau s), \tag{11}
\end{equation*}
$$

where $\tau_{\min }=4 \mathrm{~m}^{2} / \mathrm{s}$ and $\hat{s}=\tau s$. The energy spectrum of the high energy photons obtained through Compton backscattering of laser photons on the high energy electron beam has the form

$$
\begin{equation*}
f_{\gamma}(y)=\frac{1}{1.84}\left(1-y+\frac{1}{1-y}-\frac{4 y}{\xi(1-y)}+\frac{4 y^{2}}{\xi^{2}(1-y)^{2}}\right), \tag{12}
\end{equation*}
$$

with $\xi=4.8$. For $\sqrt{s_{e e}}=2 \mathrm{TeV}$, which corresponds to ${\sqrt{\hat{s}_{\gamma \gamma}}}^{\max }=0.83 \quad \sqrt{s_{e e}}=1.66 \mathrm{TeV}$, we obtain $\sigma\left(\gamma \gamma \rightarrow \ell_{4}^{+} \ell_{4}^{-}\right)=21 \mathrm{fb}, \sigma\left(\gamma \gamma \rightarrow d_{4} \bar{d}_{4}\right)=0.8 \mathrm{fb}$ and $\sigma\left(\gamma \gamma \rightarrow u_{4} \bar{u}_{4}\right)=12 \mathrm{fb}$. With $\mathscr{L}_{\gamma \gamma}=10^{34} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ this gives $2100 \ell_{4}^{+} \ell_{4}^{-}, 80 d_{4} \bar{d}_{4}$ and $1200 u_{4} \bar{u}_{4}$ pairs per year, respectively.

Fourth family quarks will be copiously produced at future pp machines. For example, at the Large Hadron Collider one expects $5000 d_{4} \bar{d}_{4}$ and $u_{4} \bar{u}_{4}$ per year [10]. A comparatively smaller number of fourth family quarks, but with a clearer background, will be produced at future TeV energy $\gamma p$ colliders [11]. For example, the LHC + TESLA proposal with $\mathscr{L}_{p p}=5 \times 10^{32} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ will produce $400 d_{4} \bar{d}_{4}$ and $1500 u_{4} \bar{u}_{4}$ pairs per year [12].

Decays of the fourth family fermions. According to the DMM approach considered here, the fourth family fermion masses are close to each other and equal to $4 g \eta$ with great accuracy. As a consequence, the dominant decay modes will be the following:

$$
\nu_{4} \rightarrow \tau^{-}+W^{+}, \quad \ell_{4}^{-} \rightarrow \nu_{\tau}+W^{-}, \quad u_{4} \rightarrow b+W^{+}, \quad d_{4} \rightarrow t+W^{-}
$$

The last decay will be followed by $t \rightarrow b+W^{+}$. Therefore, pair production of $u_{4}$ quarks will appear in the detector as two high energy $b$ jets associated with a $W^{+} W^{-}$pair. In pair production of $d_{4}$ there is an additional $W^{+} W^{-}$ pair.

Since the first and second family masses are small, small deviations in the fourth family fermion masses will also allow three body decays as $\nu_{4} \rightarrow \ell_{4}^{-}+e^{+}+\nu_{e}$ and decays like $\nu_{4} \rightarrow \ell_{4}^{-}+\pi^{+}$, etc.

In conclusion, the existence of the fourth family may manifest itself in the measurement of the $\overline{f b W}$ vertex strength.

This vertex can be investigated via the process $\gamma e \rightarrow t b \nu$ [13] at relatively low energy $\gamma e$ colliders, for example, at a $\gamma e$ machine based on the DLC $e^{+} e^{-}$linac with $\sqrt{s}=0.5 \mathrm{TeV}$. The investigation of the processes $\gamma u \rightarrow t \bar{b} d$ and $\gamma d \rightarrow \boldsymbol{t b} u$ at future $\gamma p$ colliders may provide a different way for the measurement of the strength of this vertex.

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