COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES DE L'UNIVERSITÉ D'ANKARA

Série B : Chimie

TOME : 30 ANNÉE : 1984

The Feedback And Feedforward Control of The Five CSTR Cascade Under The Effect of The Sinus Input

by

Mustafa ALPBAZ Ayşe SELEK

10

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences
de l'Université d'Ankara

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les diciplines scientifiques représentées à la Faculté des Sciences de l'Université d'Ankara.

La Revue, jusqu'à 1975 à l'exception des tomes I, II, III etait composée de trois séries

Série A : Mathématiques, Physique et Astronomie,
Série B : Chimie,
Série C : Sciences Naturelles.

A partir de 1975 la Revue comprend sept séries:

Série A_1: Mathématiques,
Série A_2: Physique,
Série A_3: Astronomie,
Série B : Chimie,
Série C_1 : Géologie,
Série C_2 : Botanique,
Série C_3 : Zoologie.

A partir de 1983 les séries de C_2 Botanique et C_3 Zoologie ont été réunies sous la seule série Biologie C et les numéros de Tome commencerons par le numéro 1.

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté des Sciences de l'Université d'Ankara. Elle accepte cependant, dans la mesure de la place disponible les communications des auteurs étrangers. Les langues Allemande, Anglaise et Française seront acceptées indifféremment. Tout article doit être accompagné d'un resumé.

Les articles soumis pour publications doivent être remis en trois exemplaires dactylographiés et ne pas dépasser 25 pages des Communications, les dessins et figures portés sur les feuilles séparées devant pouvoir être reproduits sans modifications.

Les auteurs reçoivent 25 extraits sans couverture.

l'Adresse : Dergi Yayın Sekreteri
Ankara Üniversitesi,
Fen Fakültesi,
Beşevler-Ankara
TURQUIE

The Feedback And Feedforward Control of The Five CSTR Cascade Under The Effect of The Sinus Input

Mustafa ALPBAZ* **Ayşe SELEK****

* University of Ankara, Faculty of Science, Dept, of Chemical Engineering.
** University of Anadolu, Engineering Faculty, Dept. of Chemical Engineering.

(Received January 3, 1984; Accepted June 6, 1984)

SUMMARY

In this work, the effects of the sinus changes given to the flow rate of the cascade of five continuous flow stirred tank reactors to the output variables were investigated theoretically. In addition, the feedback and feedforward control of the same cascade under the similar changes were investigated. From the results of calculations, the stability of the system was examined with the phase-plane analysis. Laplace transformations of the first and fifth tanks were derived and than related transfer functions were calculated, Bode and Nyquist diagrams were given for the same tanks with the aid of these functions.

INTRODUCTION

Alpbaz (1), has showed in his experimental work that there was ideal well stirred behaviour in the five continuous flow stirred tank reactors (CSTR) which were used in this research and the output temperature was controlled with feedforward and feedback control mechanism under the effect of step change given to the feed flow rate.

In dynamic and control work, the sinus changes were used for the stability analysis. For this analysis a lot of work dealing the effects of the sinus change on the processes have been done in literature (2,3,4) In chemical engineering, sinus changes can be given separately on the feed flow rate or input temperature.

The mathematical form of the sinus input is shown below.

$$t = o \qquad f(t) = o \tag{1}$$
$$t = t \qquad f(t) = A\sin wt$$

The sinus input which has amplitude, A, frequency, w, is shown in Fig. 1. The Laplace Transform of the sinus change,

$$L\ \{f(t)\} = \frac{wA}{s^2 + w^2} \tag{2}$$

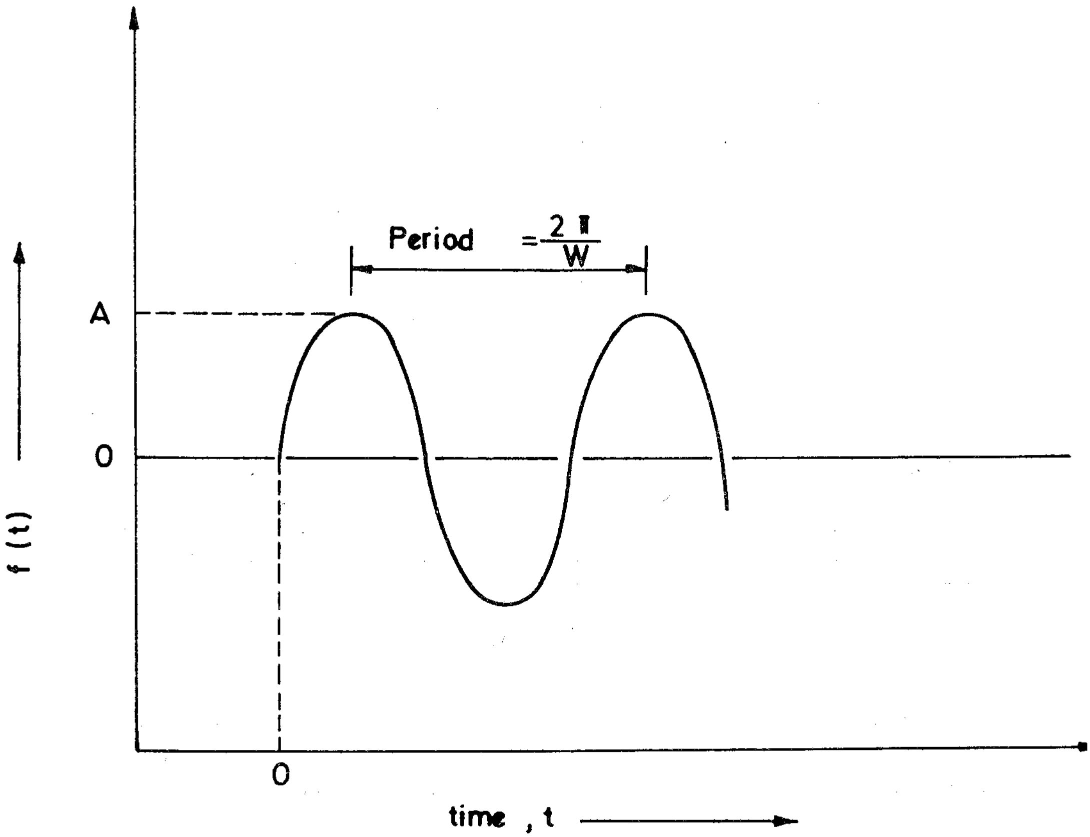

Fig. 1. The sinus change with amplitude A and frequency w.

MATHEMATICAL MODELS

Some assumptions have been done for developing the mathematical models.

1– First order reaction is taking place in reactor.

2– Heat loss from reactor is negliable.

3– Heat transfer coefficient is changed with cooling flowrate.

4– The physical properties of the reactor content are constant during transient period.

CSTR cascade model for a first order reaction.

The transient mass balance for stage 1 is.

$$\frac{dc_1}{dt} = \frac{V_1}{V} (c_0 - c_1) - k(T_1)c_1 \quad (3)$$

The transient energy balance for stage 1 is

$$\frac{dT_1}{dt} = \frac{V_1}{V} (T_0 - T_1) - \frac{V_2 \rho_2 c_{p2}}{\rho_1 V c_{p1}} (\theta_5 - \theta_4) - \frac{\Delta H c_1}{\rho_1 c_{p1}} k(T_1) \quad (4)$$

The transient energy balance for coolant

$$V_2 \rho_2 c_{p2} \theta_4 + UA (T_1 - \theta_5) = V_2 \rho_2 c_{p2} \theta_2 + M_c c_{p2} \frac{d\theta_5}{dt} \quad (5)$$

Since the time constant so small, the accumulation term in the coolant circuit can be neglected and hence the solution of equation becomes.

$$\theta_4 = T_1 - (T_1 - \theta_5) \exp (UA / V_2 \rho_2 c_{p2}) \quad (6)$$

The steady-state mass and energy balance for stage 1 are

$$\frac{dc_1}{dt} = 0 \qquad \frac{dT_1}{dt} = 0$$

$$c^\circ_1 = \frac{c_0}{1 + \frac{k(T^\circ_1)V}{V_1}} \quad (7)$$

$$T^\circ_1 = T_0 - \frac{V_2 \rho_2 c_{p2}}{\rho_1 V_1 c_{p1}} (\theta^\circ_5 - \theta^\circ_4) - \frac{k(T^\circ_1)V \Delta H c_1^\circ}{\rho_1 V_1 c_{p1}} \quad (8)$$

THE LAPLACE TRANSFORM AND TRANSFER FUNCTIONS OF THE ENERGY AND MASS BALANCE

For the examination of the dynamic and control properties of the five CSTR cascade under the effect of the sinus input, the transfer functions of the mass and energy blance were found with the aid of Laplace transform. These functions were solved putting $s = iw$ and than Bode and Nyquist diagrams were drawn.

For the first reactor, steady–state mass balance (7) was substracted from the unsteady–state balance (1) and than nonlinear terms, V_1C_1 and $k(T_1)C_1$, were linearized with Taylor's theorem and perturbation vaiables were defined.

$$\left(\frac{V}{V^\circ_1 + Vk(T^\circ_1)}\right) \cdot \frac{dc'_1}{dt} = \left(\frac{c_0 - c^\circ_1}{V^\circ_1 + Vk(T^\circ_1)}\right) V'_1 - c'_1$$

$$- \left(\frac{V\ c^\circ_1\, e^{36 \cdot 49 - 12100/T^\circ_1}\ 12100 / T_1{}^{\circ 2}}{V^\circ_1 + Vk(T_1{}^\circ)}\right) T'_1 \qquad (9)$$

$$K\ \frac{dc'_1}{dt} = L\ V'_1 - c'_1 - MT'_1 \qquad (10)$$

Laplace transform of the equation (10) is evaluated and than becomes,

$$c'_1(s) = \frac{L}{Ks + 1} V'_1(s) - \frac{M}{Ks + 1} T'_1(s) \qquad (11)$$

The similar procedure is repeated for temperature, T_1, equation (8) is substructed from equation (4) and it is written as an perturbation variables and than the nonlinear term is linearized,

$$V\rho_1 c_{p1} \frac{dT'_1}{dt} = V'_1(\rho_1 c_{p1} T_0 - \rho_1 c_{p1} T^\circ_1) - [\rho_1 c_{p1} V^\circ_1 + \Delta HV \qquad (12)$$

$$12100 / T_1{}^{\circ 2}\, c^\circ_1 e^{36 \cdot 49 - 12100 T^\circ_1} + UA]\ T'_1 + UA\theta'_5\ -(\Delta HVk(T^\circ_1))c_1{}'$$

$$N_1\ \frac{dT'_1}{dt} = N_2\ V'_1 - T'_1 + N_3\theta_5{}' - N_4 c'_1 \qquad (13)$$

Laplace transform of the equation (13) is evaluated and than putting inorder it becomes,

$$T'_1(s) = \frac{N_2}{N_1 s + 1} V'_1(s) + \frac{N_3}{N_1 s + 1} \theta'_5(s) - \frac{N_4}{N_1 s + 1} c'_1(s) \qquad (14)$$

If similar procedure is repeated for equations (5,6),

$$\frac{M_c c_{p2}}{UA + V_2\rho_2 c_{p2}} \cdot \frac{d\theta'_5}{dt} = \frac{UA}{UA + V_2\rho_2 c_{p2}} T'_1 - \theta'_5 \qquad (15)$$

If Laplace transform is evaluated,

$$\theta'_5(s) = \frac{R_2}{R_1 s + 1} T'_1(s) \tag{16}$$

If equation (16) is put in to equation (14),

$$T'_1(s) = \frac{N_2(R_1 s + 1)}{(N_1 s+1)(R_1 s+1) - N_3 R_2} V'_1(s) - \frac{N_4(R_1 s + 1)}{(N_1 s+1)(R_1 s+1) - N_3 R_2} c'_1(s) \tag{17}$$

$$c'_1(s) = \frac{L(N_1 s+1)(R_1 s+1) - LN_3 R_2 - MN_2(R_1 s+1)}{(Ks+1)(N_1 s+1)(R_1 s+1) - N_3 R_2(Ks+1) - MN_4(R_1 s+1)} V'_1(s) \tag{18}$$

$$\frac{c'_1(s)}{V'_1(s)} = \frac{L(N_1 s+1)(R_1 s+1) - LN_3 R_2 - MN_2(R_1 s+1)}{(Ks+1)(N_1 s+1)(R_1 s+1) - N_3 R_2(Ks+1) - MN_4(R_1 s+1)} \tag{19}$$

Equation (19) are used for obtaining Bode and Nyquist diagrams putting $s = iw$ for output concentrations and solved numerically. Related amplitude ratio $|G|$ and phase angle φ are given below,

$$c'_1(s) = \frac{9.625 \times 10^{-8} s^2 + 1.125 \times 10^{-7} s + 2.549 \times 10^{-10}}{s^3 + 1.180\, s^2 + 2.100 \times 10^{-2} s + 1.098 \times 10^{-4}} V'_1(s) \tag{20}$$

putting $s = iw$ and than,

$$|G| = \sqrt{\left(\frac{1.048 \times 10^{-9} w^4 + 2.053 \times 10^{-9} w^2 + 2.801 \times 10^{-14}}{w^6 + 1.351\, w^4 + 1.818 \times 10^{-4} w^2 + 1.207 \times 10^{-8}}\right)^2 + \left(\frac{-9.625 \times 10^{-8} w^5 - 1.305 \times 10^{-7} w^3 + 7.012 \times 10^{-12} w}{w^6 + 1.351\, w^4 + 1.818 \times 10^{-4} w^2 + 1.207 \times 10^{-8}}\right)^2} \tag{21}$$

$$\tan \varphi = \frac{-9.625 \times 10^{-8} w^5 - 1.305 \times 10^{-7} w^3 + 7.012 \times 10^{-12} w}{1.048 \times 10^{-9} w^4 + 2.053 \times 10^{-9} w^2 + 2.801 \times 10^{-14}} \tag{22}$$

DIGITAL COMPUTER SOLUTION

For the cascade model, steady–state solutions were obtained and used as initial values for integration of their respective models. The dynamic characteristics of cascade models without control and on addition of feedback and feedforward control were investigated. In steady–state model the input conditions must be known and in the case of countercurrent coolent flow, its input value is known when $Z = L$ and the method of solution must be able to deal with the calculation of the output value of coolant boundary conditions at the two ends of the system obtained in this situation by an iterative procedure. Newton–Raphson iterative method and Golden–Section Search were used for the solution of steady–state cascade model. The equations (3,4) described the unsteady–state cascade models can be integrated by the 4 the order Runge–Kutta method. In order to control the cascade, feedback, feedforward control, measement and valve subroutines were introduced into the programs.

The total sistem is shown in Fig. 2. The range of parameters in this work are shown below.

Rate constant,

The reaction chosen was first order and irreversible with a temperature dependent rate constant assumed to be expressible as

$$k = Ze^{-\frac{E}{RT}} \tag{23}$$

The values chosen for Z and E were taken Aris and Amundson (5)

$Z = 6.3x10^{15}$ $E = 24000$ Cal / mol

if this equation was rearranged,

$k = \exp\,(36.49 - 12100 / T)$

The value of $\Delta H = -18600$ cal / mol was selected.

The each volume of the five CSTR cascade used for simulation was the value of 3,375 cm^3.

The value of UA was calculated theoretically and compared with experimental result (1). The values of UA are given in Fig. 3.

The steady–state input conditions of five CSTR is shown in Table 1.

RESULT

When the five CSTR cascade came to the steady–state having input conditions given in Table. 1, the temperature and concentration profiles are shown in Fig. 4.

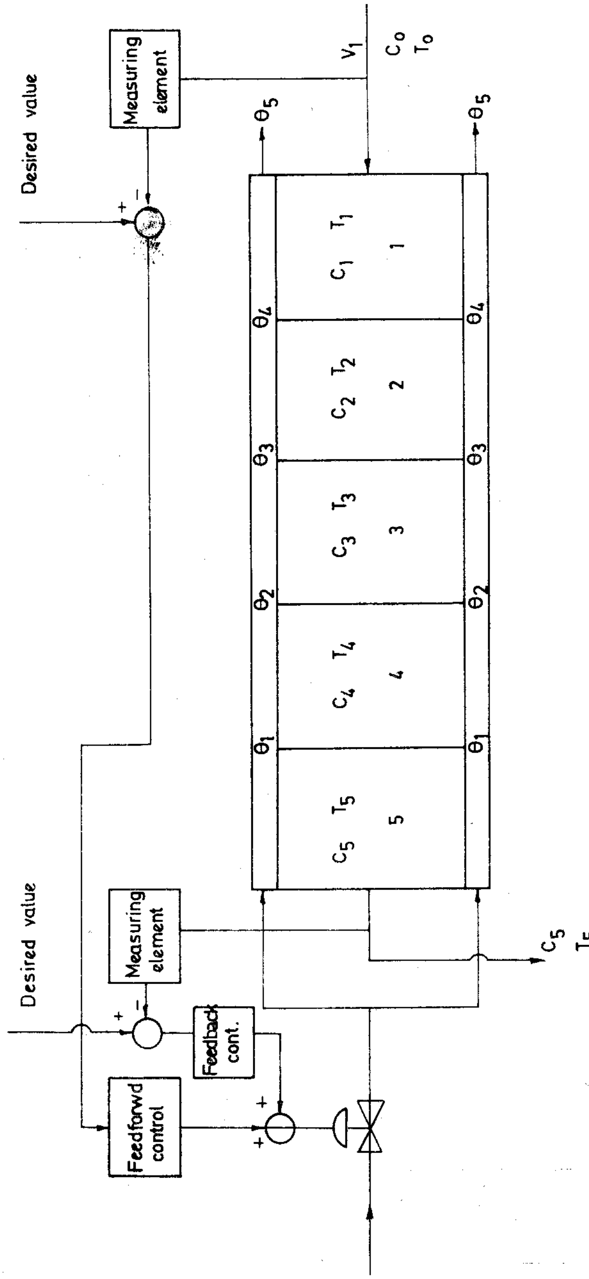

Fig. 2. Feedforward and feedback control of the five CSTR cascade.

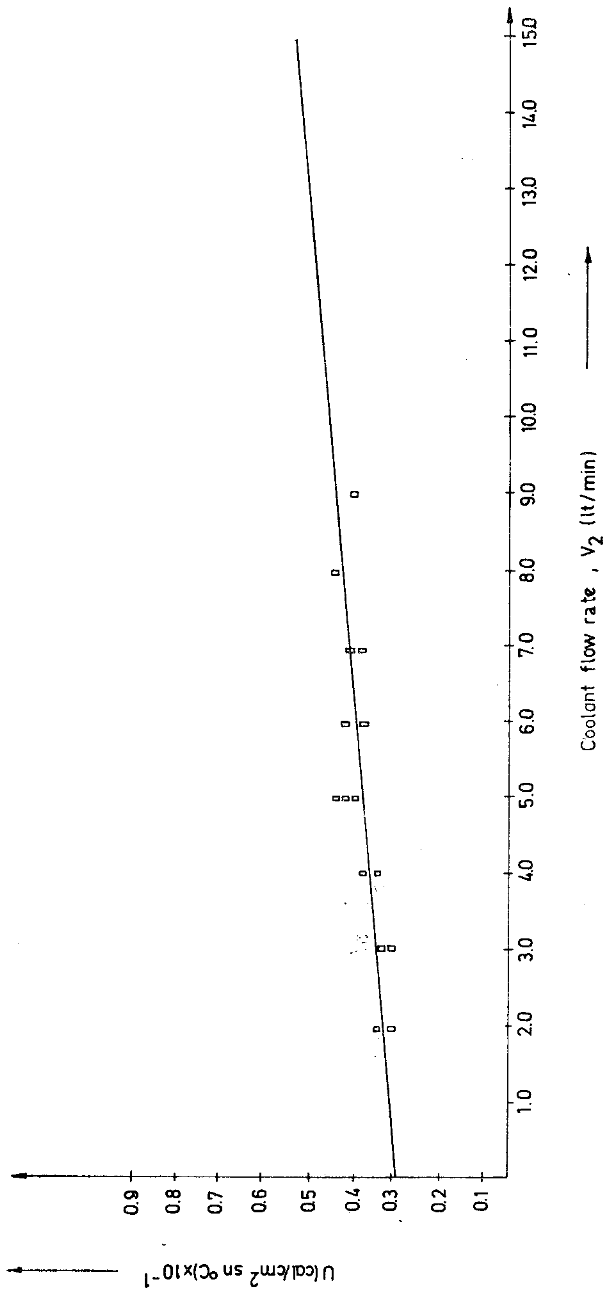

Fig. 3. The variation of the heat transfer coefficient with cooling water flow rate

Table 1. The operating steady-state conditions for the five CSTR cascade.

V_1 lt / min	V_2 lt / min	C_0 mol / lt	θ_0 °C	T_0 °C
1.0	6.0	0.5	0.5	23.0

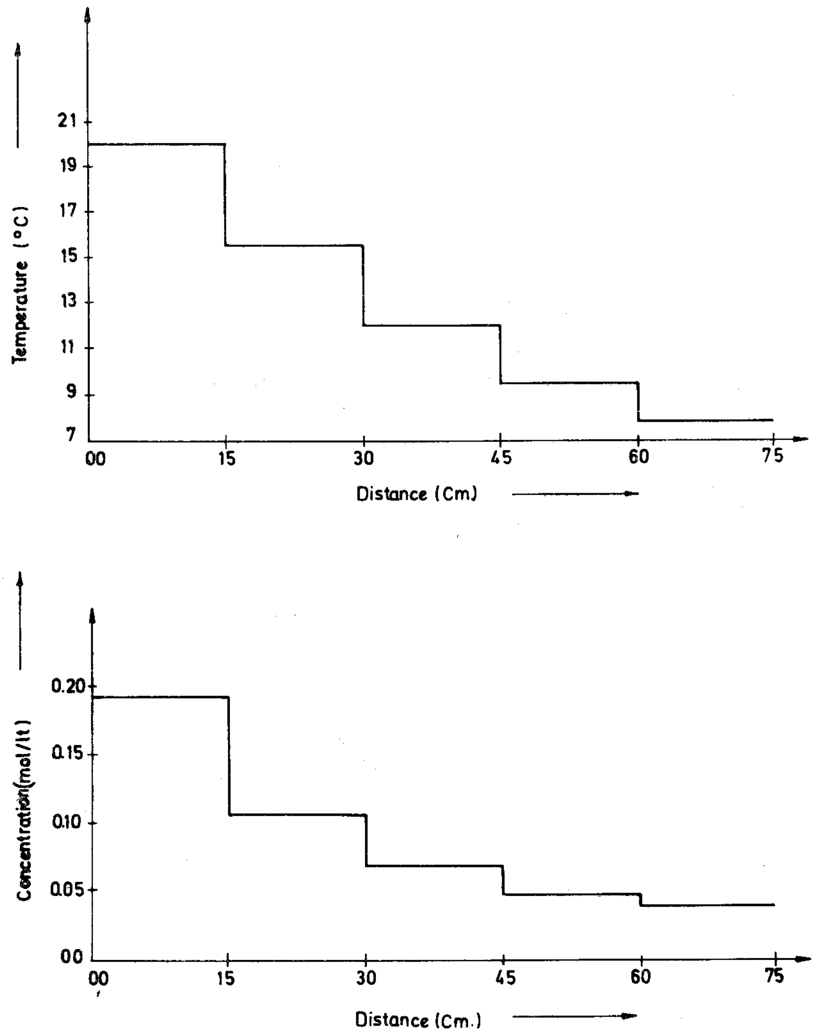

Fig. 4. Steady-state temperature and concantration profiles for the five CSTR cascade.

When the system was in the steady-state condition given in Fig. 4. the sinus changes with different frequencies were given to the feed flow rate. The results for ($w = 2\pi t / 360$) was chosen as an typical example. The time response of the output temperature which is the result of the unsteady-state digital computer solution are given in Figs. 5,6.

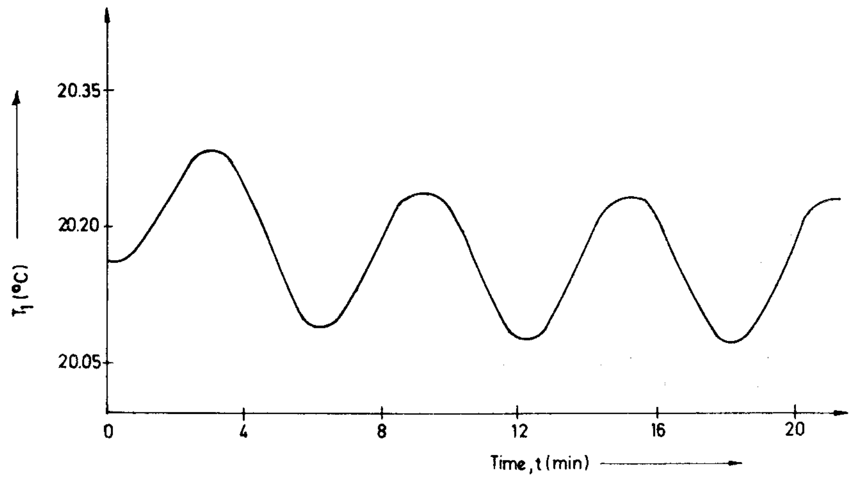

Fig. 5. Open loop dynamic temperature profiles for the first tank of the five CSTR cascade ($w = 2\pi t / 360$).

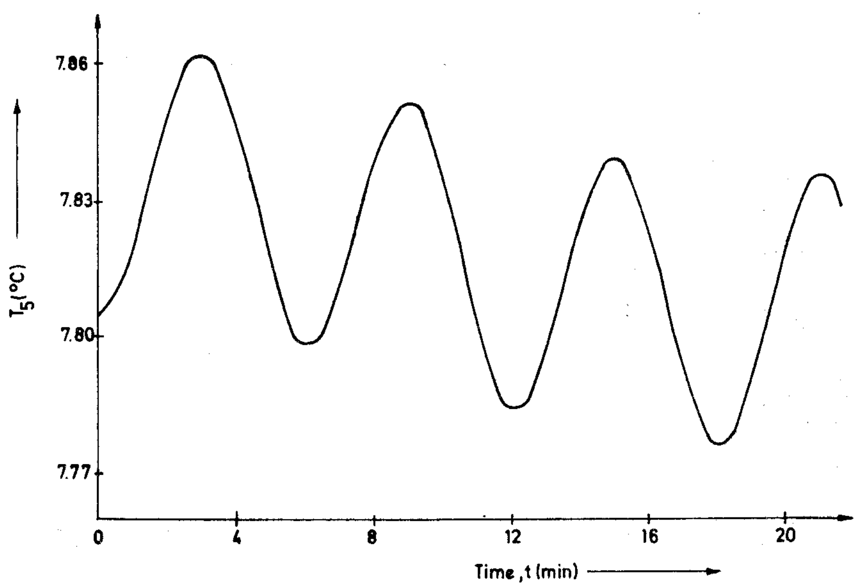

Fig. 6. Open loop dynamic temperature profiles for the fifth tank of the five CSTR cascade ($w = 2\pi t / 360$).

Related Bode diagrams for output concentrations are given in Figs. 7,8. According to the Bode stability criterion,a control system is unstable if the open–loop frequency response exhibits on the value of amplitude ratio of the function $|G|$ is less than unity at the frequency for which the phase angele φ is 180° and than it can be seen that the five CSTR cascade controlable and stable with the variation of concentration respect with time.

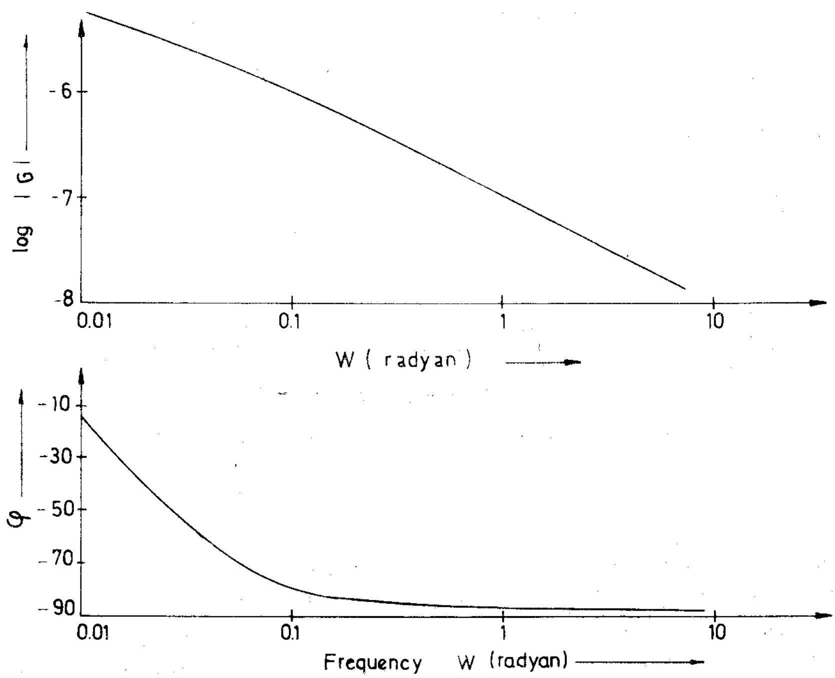

Fig. 7. Bode diagrams for the output cocentration of the first tank.

For the phase–plane analysis, the related diagrams for output concentrations and temperature effected with sinus input ($w = \pi t / 360$) are given in Figs. 9,10.

The method of the Nyquist stability criterion was examined for another stability analysis. Nyquist diagrams for output concentration

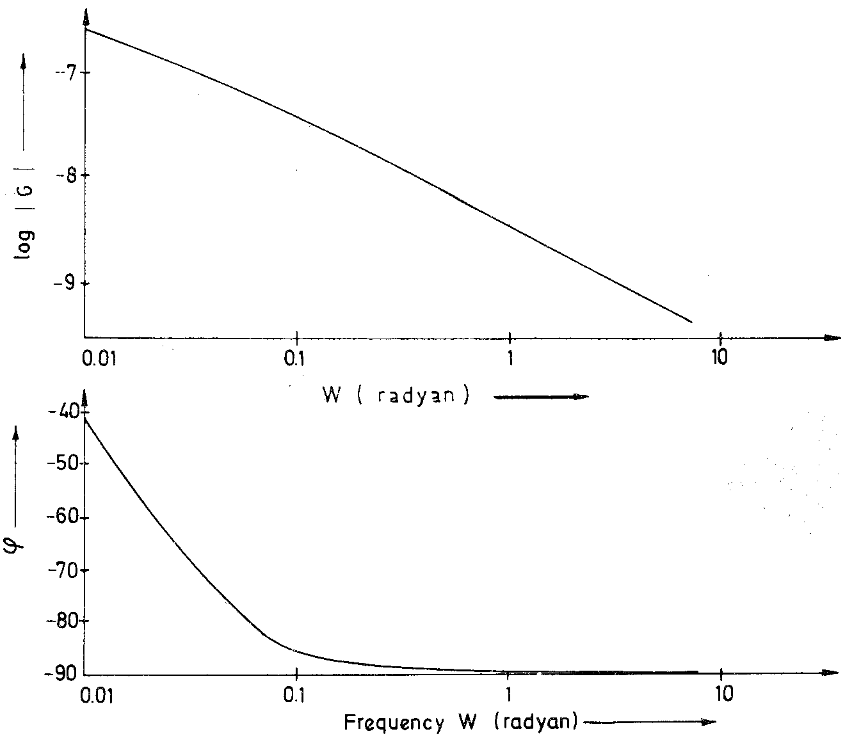

Fig. 8. Bode diagrams for the output concentration of the fifth tank.

are shown in Figs. 11,12. For this stability analysis, same amplitude ratio and phase angle shown with the equations (20,21) were used. As it is shown from Figs. 11,12, when the diagrams do not pass the right half of the real axis the variation of output concentration with time is stable according to the Nyquist stability criterion.

As a result of dynamic studying, the variation of output variables with time show stability when the sinus chance ($w = 2\pi t / 360$) effects to the system.

On the effect of sinus change ($w = 2\pi t / 360$) given to the feed flow rate of the similar system, the effect of the feedforward and feedback control system on the output temperature is shown in Fig. 13.

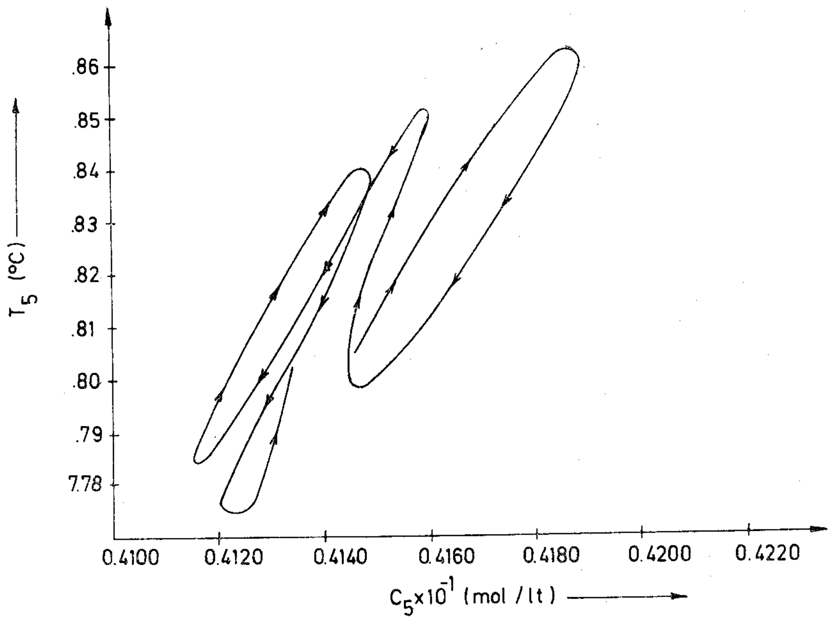

Fig. 9. Phase-plane diagram for the first tank of the five CSTR cascade (w = 2πt / 360)

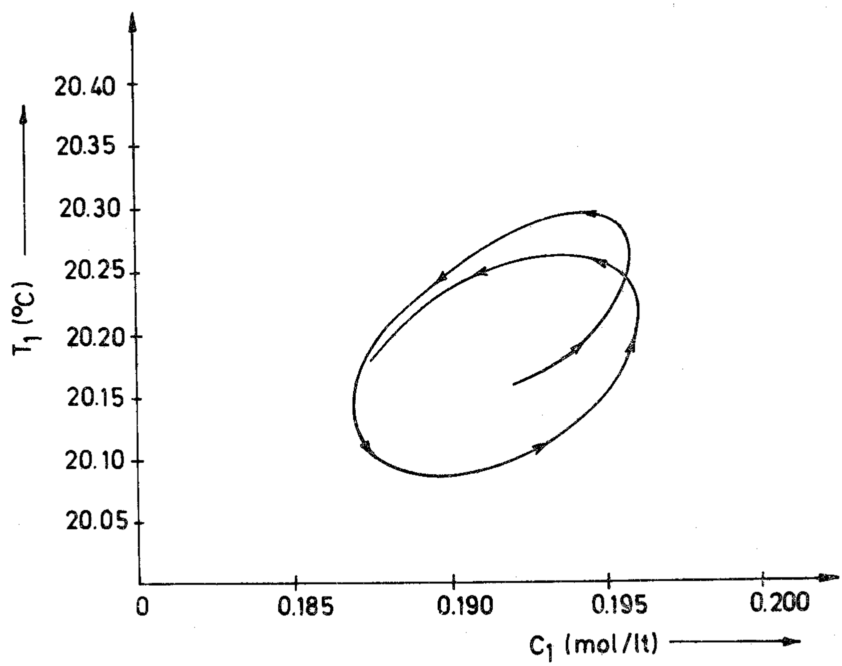

Fig. 10. Phase-plane diagram for the fifth tank of the five CSTR cascade (w = 2πt / 360).

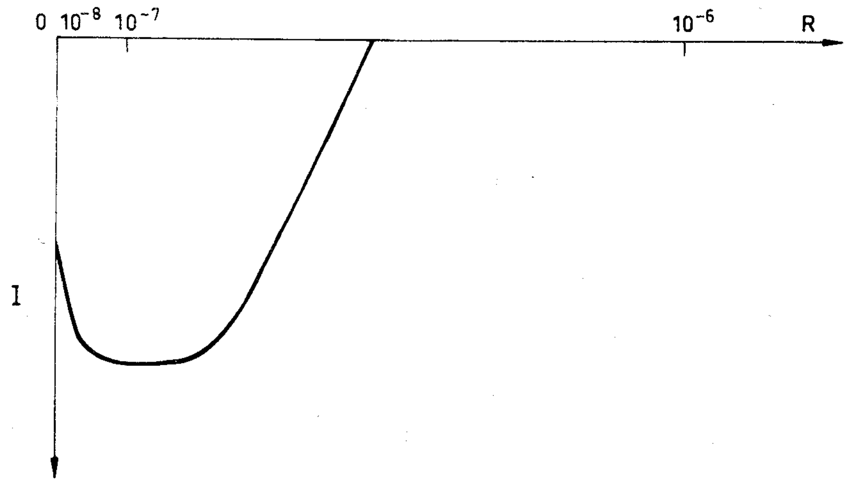

Fig. 11. Nyquist diagram for the output concentration of the first tank.

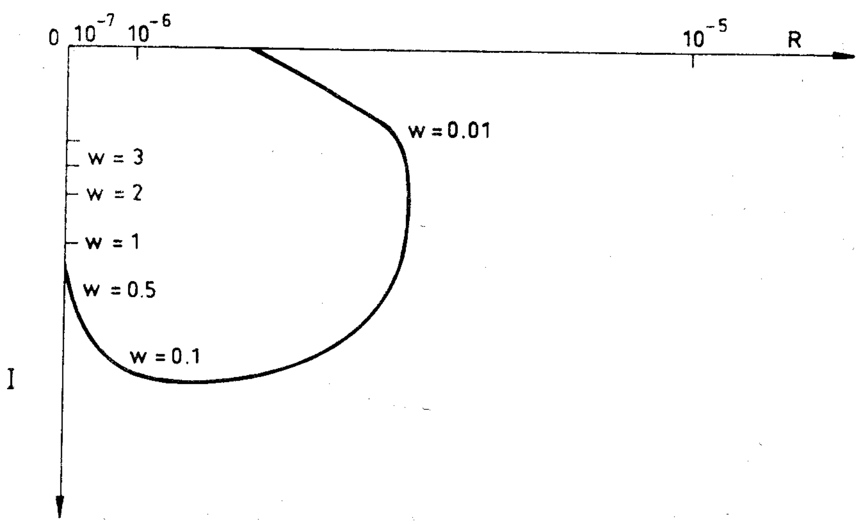

Fig. 12. Nyquist diagram for the output concentration of the fifth tank.

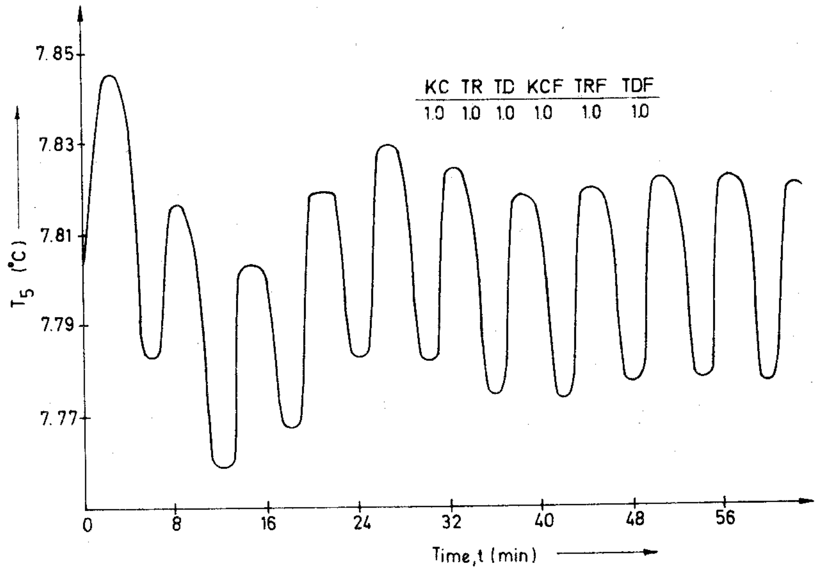

Fig. 13 Feedforward and feedback control of the fifth tank of cascade (w=2πt/3600).

Phase–plane diagram is drawn for examining the stability of the control systems and five CSTR. The related diagram is shown in Fig. 14. From the direction of the arrows it can be seen that the output temperature shows stability.

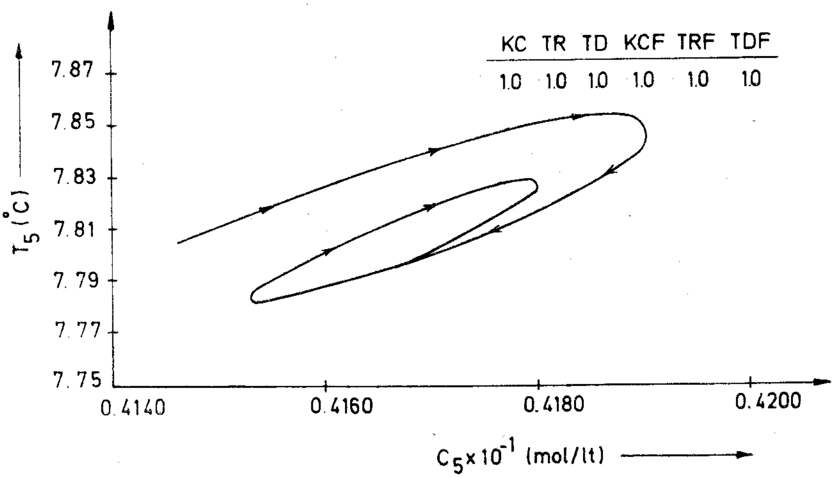

Fig. 14. Phase-plane diagram for the feedforward and feedback control of the fifth tank of cascade (w = 2πt / 360).

CONCLUSION

The variation of output temperature and concentration with time were stable when the sinus change given to the feed flow rate of the five CSTR. With the aid of Laplace trasform and trasfer functions, Bode and Nyquist diagrams were drown for first and fifth tanks and output variables are stable according to these two stability cariterions.

The output temperature, T_5, effected with sinus changes can be controllable with the feedforward and feedback control mechanisms. For the stability analysis, the phase–plane diagram was used and it was seen that system was stable.

NOMENCLATURE

A	Amplitude, Reactor heat transfer surfece (cm^2).
c_o	Initial input concentration (mol / lt).
c_n	Concentration of n'th tank (mol / lt).
c_{p1}	Spesific heat of feed (cal / g °C).
cp_2	Spesifik heat of coolant (cal / g °C).
E	Activation energy
ΔH	Heat generation by reaction (cal / mol).
k	Reaction rate constant (1 / min).
K	Proportional acting factor for feedback control.
KCF	Proportional acting factor for feedforward control.
M	Total mass holdup for the cooling jacket (g).
R	Ideal gas constand.
s	Laplace operator.
t	Time
T_0	Input temperature of reactor (°C).
T_n	Temperature of n'th stage of reactor (°C).
T_D	Derivative action time for feedback control.
TDF	Derivative action time for feedforward control.

T_R Reset time for feedback conrol.

TRF Reset time for feedforward control.

U Heat transfer coefficient (cal / cm^2 sec °C).

V Volume of each vessel (cm^2).

V_1 Feed flow rate (lt / min).

V_2 Cooling flow rate (lt / min).

θ_o Initial temperature of input coolant (°C).

θ_n Temperature of coolant in n'th tank (°C).

ρ_1 Density of reactor content (g / cm^3).

ρ_2 Density of coolant (g / cm^3).

w Frequency.

ÖZET

Bu çalışmada, beş tam karıştırmalı akım reaktörleri dizisinin besleme akış hızına verilen sinüs değişimlerinin çıkış değişkenleri üzerindeki etkileri kurulmsal olarak incelenmiştir. Ayrıca aynı sistem benzer etkiler altında iken, ileri ve geri beslemeli kontrol mekanizmalarının etkileri araştırılmıştır. Bilgisayar ile çözüm sonuçlarında, faz-düzlem analizi yönteminin yardımı ile sistemin kararlılığı incelenmiştir, ayrıca birinci ve beşinci tanklar için Laplace dönüşümü yapılarak ilgili taransfer fonksiyonları hesaplanmıştır. Bu fonksiyonlar yardımı ile aynı tanklar için Bode ve Nyquist diyagramları çizilmiştir.

REFERENCES

1- **Alpbaz M.** Ph. D. Thesis, University of Aston (1975).

2- **Luyben W.L.** Process Modelling, Simulation and Control for Chemical Engineers, Mc-Graw Hill, Kogakusha (1973).

3- **Gutierrez, H.J.. and Cooper. A.R.** Chem. Eng. J. vol. 17, 13. (1978).

4- **Gutierrez, H.J.. and Cooper, A.R.** Chem. Eng. J. vol. 17, 19. (1978).

5- **Aris. R.. and Amundson, N.R.** Chem. Eng. Sci. vol. 7, 121. (1953).