

ON THE FOCAL SURFACES OF THE CONGRUENCES GENERATED BY THE INSTANTANEOUS SCREWING AXES CONNECTED WITH SOME SURFACES

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ABSTRACT

In this paper, the focal surfaces of the congruences derived in [1] and [3] have been investigated and correspondences between them have been explained.

1. INTRODUCTION

Let a surface \vec{x} be referred to its lines of curvatures. The congruences generated by the instantaneous screwing axes \vec{G}, \vec{G}^* of the moving trihedrons connected with these lines are respectively,

$$\left\{ \begin{array}{l} \vec{y} = \vec{r} + t\vec{g}, \quad \vec{r} = \vec{x} + \frac{1}{r} \vec{s} \\ \vec{y}^* = \vec{r} + t^*\vec{g}^*, \quad \vec{r} = \vec{x} + \frac{1}{r} \vec{s} \end{array} \right. \quad (1.1)$$

[1]. In case \vec{y} and \vec{y}^* are normal congruences, let the surfaces generating these, be \vec{z} and \vec{z}^* . And let these surfaces be referred to their lines of curvature. The congruence generated by the instantaneous screwing axis \vec{G}^* of the moving trihedron connected with the lines of curvature $u = \text{const.}$ of \vec{z} are

$$\vec{y}^* = \vec{r} + t^* \vec{g}^*, \quad \vec{r} = \vec{z} + \frac{1}{b} \vec{n}. \quad (1.2)$$

And the congruence generated by the instantaneous screwing axis \vec{G}^{**} of the moving trihedron connected with the lines of curvature $v = \text{const.}$ of \vec{z} are

$$\vec{y}^{**} = \vec{r} + t^{**} \vec{g}^{**}, \quad \vec{r} = \vec{z} + \frac{1}{\beta} \vec{n} \quad (1.3)$$

[3].

2. THE PROPERTIES OF THE FOCAL SURFACES OF THE CONGRUENCES $\vec{y}, \vec{y}^*, \vec{y}^{**}$

Since \vec{p}, \vec{k} are the focal surfaces of the congruence \vec{y} ; \vec{p}, \vec{k} of \vec{y}^* ; \vec{p}^*, \vec{k}^* of \vec{y}^* ; and $\vec{p}^{**}, \vec{k}^{**}$ of \vec{y}^{**} [3], to investigate considering the cases where they coincide and refer to their lines of curvature, first we may write the moving trihedrons (DARBOUX's trihedrons) connected with a common point before calculating their first and second fundamental forms.

1) Since the moving trihedron connected with the point \vec{x} of the line of curvature $v = \text{const.}$ on the surface $\vec{x}(u, v)$ is $(\vec{x}_1, \vec{x}_2, \vec{\xi})$, the trihedrons connected with the focal points corresponding to ρ_{II} of the focal surfaces $\vec{p}, \vec{p}^*, \vec{k}^{**}$ belonging to the congruences $\vec{y}, \vec{y}^*, \vec{y}^{**}$ and coinciding with the center surface \vec{r} of the surface \vec{x} , are respectively,

$$(\vec{\xi}, \vec{x}_2, -\vec{x}_1), \quad (\vec{\xi}, \vec{x}_2, -\vec{x}_1), \quad (-\vec{\xi}, -\vec{x}_2, -\vec{x}_1).$$

2) Since the moving trihedron connected with the point \vec{x} of the line of curvature $u = \text{const.}$ on the surface $\vec{x}(u, v)$ is $(\vec{x}_2, -\vec{x}_1, \vec{\xi})$, the trihedrons connected with the focal points corresponding to ρ_{II} of the focal surfaces $\vec{p}, \vec{k}^*, \vec{p}^{**}$ belonging to the congruences $\vec{y}, \vec{y}^*, \vec{y}^{**}$ and coinciding with the center surface \vec{r} of the surface \vec{x} , are respectively,

$$(\vec{\xi}, -\vec{x}_1, -\vec{x}_2), \quad (\vec{\xi}, -\vec{x}_1, -\vec{x}_2), \quad (-\vec{\xi}, \vec{x}_1, -\vec{x}_2).$$

If we calculate the first and the second fundamental forms of the above focal surfaces

1) for the local surfaces $\vec{p}, \vec{p}^*, \vec{k}^{**}$, we find,

$$\left. \begin{aligned} E_{II} &= \bar{E}^*_{II} = \bar{E}_I^{**} = \left(\frac{1}{r}\right)_1^2 E \\ F_{II} &= \bar{F}^*_{II} = \bar{F}_I^{**} = \left(\frac{1}{r}\right)_1 \left(\frac{1}{r}\right)_2 \sqrt{EG} \\ G_{II} &= \bar{G}^*_{II} = \bar{G}_I^{**} = \left(\frac{1}{r}\right)_2^2 \frac{r^2 + q^2}{q^2} G, \end{aligned} \right\} \quad (2.1)$$

$$\begin{aligned} [I]_{II} &= [I^*]_{II} = [I^{**}]_I = \left(\frac{1}{r}\right)_1^2 E du^2 + 2 \left(\frac{1}{r}\right)_1 \left(\frac{1}{r}\right)_2 \sqrt{EG} du dv \\ &+ \left(\frac{1}{r}\right)_2^2 \frac{r^2 + q^2}{q^2} G dv^2 \end{aligned} \quad (2.2)$$

and

$$\left. \begin{aligned} L_{II} &= \bar{L}^*_{II} = \bar{L}_I^{**} = \left(\frac{1}{r}\right)_1 r E \\ M_{II} &= \bar{M}^*_{II} = \bar{M}_I^{**} = 0 \\ N_{II} &= \bar{N}^*_{II} = \bar{N}_I^{**} = \left(\frac{1}{r}\right)_2 \frac{r\bar{q}}{q} G, \end{aligned} \right\} \quad (2.3)$$

$$[II]_{II} = [\bar{II}^*]_{II} = [\bar{II}^{**}]_I = r \left[\left(\frac{1}{r}\right)_1 E du^2 + \left(\frac{1}{r}\right)_2 \frac{\bar{q}}{q} G dv^2 \right].$$

From these we may derive the below conclusion:

Conclusion: 2.1. The focal surfaces $\vec{p}, \vec{p}^*, \vec{k}^{**}$ of the congruences $\vec{y}, \vec{y}^*, \vec{y}^{**}$ are different positions of the center surface \vec{r} of the base surface \vec{x} , in space.

Also, the Gaussian and the mean curvature of these focal surfaces, we find

$$K_{II} = \bar{K}^*_{II} = \bar{K}_{I^{**}} = \frac{\bar{q}\bar{q}}{\left(\frac{1}{r}\right)_1 \left(\frac{1}{r}\right)_2} \quad (2.5)$$

and

$$H_{II} = \bar{H}_{II} = \bar{H}_{I^{**}} = \frac{\left(\frac{1}{r}\right)_1 \bar{q}\bar{q} - \left(\frac{1}{r}\right)_2 (r^2 + q^2)}{2r \left(\frac{1}{r}\right)_1 \left(\frac{1}{r}\right)_2} \quad (2.6)$$

Since, $r_1 \neq 0$, $r_2 \neq 0$ from (2.1) and (2.3) we derive the conditions $F_{II} = \bar{F}^*_{II} = \bar{F}_{I^{**}} \neq 0$ and $M_{II} = \bar{M}^*_{II} = \bar{M}_{I^{**}} = 0$. From these, the following theorem may be stated:

Theorem 2.2. Since the surface $\vec{x}(u, v)$ cannot be a canal surface or at the same time cannot be both Mulür surface and tube-shaped canal surface, the parameter curves $v = \text{const.}$ and $u = \text{const.}$ of the focal surfaces \vec{p} , \vec{p}^* , \vec{k}^{**} of the congruences \vec{y} , \vec{y}^* , \vec{y}^{**} cannot be the lines of curvature.

Since $q \neq 0$, $\bar{q} \neq 0$, $r_1 \neq 0$, $r_2 \neq 0$ in (2.5) and (2.6), we find $K_{II} = \bar{K}^*_{II} = \bar{K}_{I^{**}} \neq 0$ and $H_{II} = \bar{H}^*_{II} = \bar{H}_{I^{**}} \neq 0$.

Therefore the theorem below may be stated:

Theorem 2.3. Since the surface $\vec{x}(u, v)$ cannot be Mulür surface, canal surface or tube-shaped surface, the focal surfaces \vec{p} , \vec{p}^* , \vec{k}^{**} of the congruences \vec{y} , \vec{y}^* , \vec{y}^{**} respectively, cannot be developable surface, minimal surface.

2) For the local surfaces \vec{p} , \vec{k}^* , \vec{p}^{**} we find,

$$\left. \begin{aligned} \bar{E}_{II} = \bar{E}^*_{I} = \bar{E}^{**}_{II} &= \left(\frac{1}{\bar{r}}\right)_1^2 \frac{\bar{r}^2 + \bar{q}^2}{\bar{q}^2} E \\ \bar{F}_{II} = \bar{F}^*_{I} = \bar{F}^{**}_{II} &= \left(\frac{1}{\bar{r}}\right)_1 \left(\frac{1}{\bar{r}}\right)_2 \sqrt{EG} \\ \bar{G}_{II} = \bar{G}^*_{I} = \bar{G}^{**}_{II} &= \left(\frac{1}{\bar{r}}\right)_2^2 G, \end{aligned} \right\} \quad (2.7)$$

$$\begin{aligned} [\bar{I}]_{II} = [\bar{I}^*]_{I} = [\bar{I}^{**}]_{II} &= \left(\frac{1}{\bar{r}}\right)_1^2 \frac{\bar{r}^2 + \bar{q}^2}{\bar{q}^2} E \, du^2 + \\ 2 \left(\frac{1}{\bar{r}}\right)_1 \left(\frac{1}{\bar{r}}\right)_2 \sqrt{EG} \, dudv &+ \left(\frac{1}{\bar{r}}\right)_2^2 G \, dv^2 \end{aligned} \quad (2.8)$$

and

$$\left. \begin{aligned} \bar{L}_{II} = \bar{L}^*_{I} = \bar{L}^{**}_{II} &= \left(\frac{1}{\bar{r}}\right)_1 \frac{\bar{r}q}{q} E \\ \bar{M}_{II} = \bar{M}^*_{I} = \bar{M}^{**}_{II} &= 0 \\ \bar{N}_{II} = \bar{N}^*_{I} = \bar{N}^{**}_{II} &= \left(\frac{1}{\bar{r}}\right)_2 \bar{r} G, \end{aligned} \right\} \quad (2.9)$$

$$[\bar{\Pi}]_{II} = [\bar{\Pi}^*]_{I} = [\bar{\Pi}^{**}]_{II} = \bar{r} \left[\left(\frac{1}{\bar{r}}\right)_1 \frac{q}{q} E \, du^2 + \left(\frac{1}{\bar{r}}\right)_2 G \, dv \right]. \quad (2.10)$$

From these we may write the below conclusion:

Conclusion 2.4. The focal surfaces $\overset{\rightrightarrows}{p}$, $\overset{\rightrightarrows}{k^*}$, $\overset{\rightrightarrows}{p^{**}}$ of the congruences $\overset{\rightrightarrows}{y}$, $\overset{\rightrightarrows}{y^*}$, $\overset{\rightrightarrows}{y^{**}}$ respectively, are different positions of the center surface $\overset{\rightrightarrows}{r}$ of the base surface $\overset{\rightrightarrows}{x}$, in space.

Also, the values of K and H for these focal surfaces are found as

$$\bar{K}_{II} = \bar{K}^*_{I} = \bar{K}^{**}_{II} = \frac{\bar{q}\bar{q}}{\left(\frac{1}{\bar{r}}\right)_1 \left(\frac{1}{\bar{r}}\right)_2} \quad (2.11)$$

and

$$\bar{H}_{II} = \bar{H}_I^* = \bar{H}_{II}^{**} = \frac{\left(\frac{1}{\bar{r}}\right)_2 q\bar{q} - \left(\frac{1}{\bar{r}}\right)_1 (\bar{r}^2 + \bar{q}^2)}{2\bar{r} \left(\frac{1}{\bar{r}}\right)_1 \left(\frac{1}{\bar{r}}\right)_2}. \quad (2.12)$$

Since $\bar{r}_1 \neq 0$, $\bar{r} \neq 0$ from (2.7) and (2.9), we find the conditions $\bar{F}_{II} = \bar{F}_I^* = \bar{F}_{II}^{**} \neq 0$ and $\bar{M}^*_{II} = \bar{M}_I^* = \bar{M}_{II}^{**} = 0$.

From these we may write the below theorem:

Theorem 2.5. Since the surface $\vec{x}(u, v)$ cannot be canal surface or tube-shaped surface, the parameter curves $v = \text{const.}$ and $u = \text{const.}$ of the focal surfaces $\overset{\rightrightarrows}{p}$, $\overset{\rightrightarrows}{k^*}$, $\overset{\rightrightarrows}{p^{**}}$ of the congruences $\overset{\rightrightarrows}{y}$, $\overset{\rightrightarrows}{y^*}$, $\overset{\rightrightarrows}{y^{**}}$ cannot be lines of curvature.

Since $q \neq 0$, $\bar{q} \neq 0$, $\bar{r}_1 \neq 0$, $\bar{r}_2 \neq 0$ in (2.11) and (2.12), therefore the below theorem may be written.

Theorem 2.6. Since the surface $\vec{x}(u, v)$ cannot be Mulür surface or general cylindric surface, canal surface or tub-shaped surface, the focal surfaces $\overset{\rightrightarrows}{p}$, $\overset{\rightrightarrows}{k^*}$, $\overset{\rightrightarrows}{p^{**}}$ of the congruences $\overset{\rightrightarrows}{y}$, $\overset{\rightrightarrows}{y^*}$, $\overset{\rightrightarrows}{y^{**}}$ respectively cannot be developable surface, minimal surface.

On the other hand to investigate the focal surface \vec{k} and $\vec{k}^{\rightrightarrows}$ belonging to the congruences $\overset{\rightrightarrows}{y}$ and $\overset{\rightrightarrows}{y^*}$ respectively and coinciding with the center surface of the surfaces \vec{z} and $\vec{z}^{\rightrightarrows}$ but which do not coincide with the center surfaces \vec{r} and $\vec{r}^{\rightrightarrows}$ of the surface $\vec{x}(u, v)$, first, we may write the moving trihedrons connected with the focal point corresponding to ρ_I of \vec{k} , connected with the focal point corresponding to $\bar{\rho}_I$ of $\vec{k}^{\rightrightarrows}$ before calculating their first and second fundamental forms.

1) For the focal surface \vec{k} , from

$$\vec{k}_1 = -\left(\frac{1}{b}\right)_1 \vec{g}, \quad \vec{k}_2 = \frac{r_2 \vec{q}}{r_1 q - q_1 r} \vec{x}_1 - \left(\frac{1}{b}\right)_2 \vec{g}$$

and

$$\vec{n}_I = \frac{\vec{k}_1 \wedge \vec{k}_2}{\sqrt{(\vec{k}_1 \wedge \vec{k}_2)}} = \frac{\vec{q} x_2 - r \vec{\xi}}{\sqrt{r^2 + q^2}} = -\vec{z}_1,$$

$$\vec{n}_I \wedge (-\vec{g}) = \vec{x}_1$$

the trihedron

$$(-\vec{g}, \vec{x}_1, \vec{n}_I) \tag{2.13}$$

is found.

2) For the focal surface \vec{k} , from

$$\vec{k}_1 = -\left(\frac{1}{\beta}\right)_1 \vec{g} - \frac{\vec{r}_2 q}{q_2 \vec{r} - \vec{r}_2 q} \vec{x}_2, \quad \vec{k}_2 = -\left(\frac{1}{\beta}\right)_2 \vec{g}$$

and

$$\vec{n}_I = \frac{\vec{k}_2 \wedge \vec{k}_1}{\sqrt{(\vec{k}_2 \wedge \vec{k}_1)}} = \frac{\vec{q} x_1 - \vec{r} \vec{\xi}}{\sqrt{\vec{r}^2 + q^2}} = -\vec{z}_2,$$

$$\vec{n}_I \wedge (-\vec{g}) = \vec{x}_2$$

the trihedron is

$$(-\vec{g}, \vec{x}_2, \vec{n}_I). \tag{2.14}$$

If we calculate the first and the second fundamental forms of the focal surfaces

1) For \vec{k} , we find,

$$\left. \begin{aligned} E_I &= \left(\frac{1}{b}\right)_1^2 E \\ F_I &= \left(\frac{1}{b}\right)_1 \left(\frac{1}{b}\right)_2 \sqrt{EG} \\ G_I &= \left[\left(\frac{r_2 \bar{q}}{r_1 q - q_1 r}\right)^2 + \left(\frac{1}{b}\right)_2^2 \right] G, \end{aligned} \right\} \quad (2.15)$$

$$\begin{aligned} [I]_I &= \left(\frac{1}{b}\right)_1^2 E du^2 + 2 \left(\frac{1}{b}\right)_1 \left(\frac{1}{b}\right)_2 \sqrt{EG} dudv + \\ &\left[\left(\frac{r_1 \bar{q}}{r_1 q - q_1 r}\right)^2 + \left(\frac{1}{b}\right)_2^2 \right] G dv^2 \end{aligned} \quad (2.16)$$

and

$$\left. \begin{aligned} L_I &= - \left(\frac{1}{b}\right)_1 \frac{r_1 q - q_1 r}{r^2 + q^2} E \\ M_I &= 0 \\ N_I &= \left(\frac{1}{b}\right)_1 \frac{r^2 \bar{q}^2}{r_1 q - q_1 r} G, \end{aligned} \right\} \quad (2.17)$$

$$[II]_I = - \left(\frac{1}{b}\right)_1 \frac{r_1 q - q_1 r}{r^2 + q^2} E du^2 + \left(\frac{1}{b}\right)_1 \frac{r^2 \bar{q}^2}{r_1 q - q_1 r} G dv^2 \quad (2.18)$$

and also,

$$K_I = - \frac{q^2 (r_1 q - q_1 r)^2}{\left(\frac{1}{b}\right)_1 \left(\frac{1}{b}\right)_1 r^2 (r^2 + q^2)^2}, \quad (2.19)$$

$$\begin{aligned} H_I &= \frac{r_1 q - q_1 r}{2 \left(\frac{1}{b}\right)_1 \left(\frac{1}{b}\right)_1^2 r^4 \bar{q}^2 (r^2 + q^2)^2} \left\{ \left(\frac{1}{b}\right)_1 (r^2 + q^2) r^2 \bar{q}^2 \left[q^2 \left(\frac{1}{b}\right)_1 - \right. \right. \\ &\left. \left. - r^2 \left(\frac{1}{b}\right)_1 \right] - \left(\frac{1}{b}\right)_2^2 (r_1 q - q_1 r) q^2 \right\}. \end{aligned} \quad (2.20)$$

2) For \vec{k} ,

$$\left. \begin{aligned} \bar{E}_I &= \left[\left(\frac{1}{\beta} \right)_1^2 + \left(\frac{\bar{r}_2 \bar{q}}{\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q}} \right)^2 \right] E \\ \bar{F}_I &= \left(\frac{1}{\beta} \right)_1 \left(\frac{1}{\beta} \right)_2 \sqrt{EG} \\ \bar{G}_I &= \left(\frac{1}{\beta} \right)_2^2 G, \end{aligned} \right\} \quad (2.21)$$

$$\begin{aligned} [I]_I &= \left[\left(\frac{1}{\beta} \right)_1^2 + \left(\frac{\bar{r}_2 \bar{q}}{\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q}} \right)^2 \right] E \, du + 2 \left(\frac{1}{\beta} \right)_1 \left(\frac{1}{\beta} \right)_2 \sqrt{EG} \, du \, dv \\ &\quad + \left(\frac{1}{\beta} \right)_2^2 G \, dv^2 \end{aligned} \quad (2.22)$$

and

$$\left. \begin{aligned} \bar{L}_I &= \left(\frac{1}{\beta} \right)_2 \frac{\bar{r}_2 \bar{q}^2}{\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q}} E \\ \bar{M}_I &= 0 \\ \bar{N}_I &= \left(\frac{1}{\beta} \right)_2 \frac{\bar{r}_2 \bar{q} - \bar{q}_2 \bar{r}}{\bar{r}^2 + \bar{q}^2} G, \end{aligned} \right\} \quad (2.23)$$

$$[II]_I = \left(\frac{1}{\beta} \right)_2 \frac{\bar{r}_2 \bar{q}^2}{\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q}} E \, du^2 + \left(\frac{1}{\beta} \right)_2 \frac{\bar{r}_2 \bar{q} - \bar{q}_2 \bar{r}}{\bar{r}^2 + \bar{q}^2} G \, vd^2 \quad (2.24)$$

and also,

$$\bar{K}_I = - \frac{\bar{q}^2 (\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q})^2}{\left(\frac{1}{\beta} \right)_2 \left(\frac{1}{\beta} \right)_2 \bar{r}^2 (\bar{r}^2 + \bar{q}^2)^2}, \quad (2.25)$$

$$\bar{H}_I = \frac{\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q}}{2 \left(\frac{1}{\beta} \right)_2 \left(\frac{1}{\beta} \right)_2^2 (\bar{r}^2 + \bar{q}^2) \bar{r}^4 \bar{q}^2} \quad (2.26)$$

$$\left\{ \bar{r}^2 \bar{q}^2 (\bar{r}^2 + \bar{q}^2) \left(\frac{1}{\beta} \right)_2 \left[\left(\frac{1}{\beta} \right)_2 \bar{r}^2 - \left(\frac{1}{\beta} \right)_2 \bar{q}^2 \right] - \left(\frac{1}{\beta} \right)_2^2 (\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q})^2 \bar{q}^2 \right\}$$

are written.

1) Since the focal surface \vec{k} coincides with the center surface belonging to the lines of curvature $v = \text{const.}$ of the surface \vec{z} , that is $\vec{k} = \vec{r} + \rho \vec{g} = \vec{z} + \frac{1}{b} \vec{n} = \vec{z} - \frac{1}{b} \vec{g}$, we may write,

$$b = b_I = - [(-\vec{g}) \cdot (\vec{n}_I)] = - [\vec{n}_I \cdot (-\vec{g})] = \frac{r_1 q - q_1 r}{r^2 + q^2} \neq 0, \quad (r_1 q - q_1 r \neq 0). \quad (2.27)$$

Since $F_I \neq 0$ ($b_1 \neq 0$, $b_2 \neq 0$) in (2.15) and $M_I = 0$ in (2.17), the below theorem may be written.

Theorem 2.7. Since the surface \vec{z} cannot be canal surface or tube-shaped surface, the parameter curves $v = \text{const.}$ and $u = \text{const.}$ of the focal surface \vec{k} of the congruence \vec{y} , cannot be lines of curvature.

From (2.19) and (2.20) $K_I \neq 0$, $H_I \neq 0$ ($q \neq 0$, $r_1 q - q_1 r \neq 0$) are seen. From this the below theorem may be written.

Theorem 2.8. Since the surface \vec{x} (u, v) cannot be Mülür surface and the surface which have the lines of curvature $v = \text{const.}$ consisting of plane curves, the focal surface \vec{k} of the congruence \vec{y} , cannot be developable surface, minimal surface.

2) Since the focal surface \vec{k} coincides with the central surface belonging to the lines of curvature $u = \text{const.}$ on the surface \vec{z} that is $\vec{k} = \vec{r} + \rho \vec{g} = \vec{z} + \frac{1}{\beta} \vec{n} = \vec{z} - \frac{1}{\beta} \vec{g}$, we may write

$$\bar{\beta} = \bar{\beta}_I = - [\vec{n}_I \cdot (-\vec{g}_2)] = \frac{\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q}}{\bar{r}^2 + \bar{q}^2} \neq 0, \quad (\bar{q}_2 \bar{r} - \bar{r}_2 \bar{q} \neq 0) \quad (2.28)$$

It can be seen that at (2.21), $\bar{F}_I \neq 0$ ($\bar{\beta}_1 \neq 0, \bar{\beta}_2 \neq 0$). If we take the condition $\bar{M}_I = 0$ at (2.23) into consideration together with these, we may write the below theorem.

Theorem 2.9. Since the surface \vec{z} cannot be canal surface or tube-shaped surface, the parameter curves $v = \text{const.}$ and $u = \text{const.}$ on the focal surface \vec{k} of the congruence \vec{y} , cannot be lines of curvature.

From (2.25) and (2.26) $\bar{K}_I \neq 0, \bar{H}_I \neq 0$ ($\bar{q} \neq 0, \bar{q}_2\bar{r}-\bar{r}_2\bar{q} \neq 0$) are seen. From this below theorem may be written.

Theorem 2.10. Since the surface $\vec{x}(u, v)$ cannot be Mülür surface and the surface which have with the lines of curvature $u = \text{const.}$ consisting of plane curves, the focal surface \vec{k} of the congruence \vec{y} cannot be developable surface, minimal surface.

REFERENCE

[1] T. GÜL, On the rectilinear congruences generated by the instantaneous screwing axes connected with a surface, Bull. of Techn. Univ. İst. (1990).
 [2] T. GÜL, On the surfaces which cut ortogonally the congruences generated by the instantaneous screwing axes connected with a surface, Y.Ü.D. İstanbul (1989).
 [3] T. GÜL, On the congruences of lines generated by the instantaneous screwing axes connected with some surfaces, Communications Seri A1 Ankara (1991).
 [4] P. EISENHART, A treatise on the Differential Geometry of Curves and Surfaces, New York, (1960).