Commun. Fac. Sci. Univ. Ank. Ser. A_1
V. 38, Number 1-2, pp 67-76 (1989)

ON THE COHOMOLOGY GROUPS OF COMPLEX ANALYTIC MANIFOLDS

SABAHATTİN BALCI

Dept. of Mathematics, Faculty of Science, Ankara University, ANKARA

SUMMARY

Let X be a connected complex analytic manifold of dimension n with fundamental group $H_x \neq \{1\}$, for any $x \in X$. Let H be the sheaf of the fundamental groups over X, $[H,H] \subset H$ be the commutator subsheaf, Q be the sheaf of Abelian groups [1] determined by [H,H] over X and A be the Restricted sheaf of germs of holomorphic functions on X defined in [4]. It is shown, in this paper, that; The Cohomology group $H^o(X,Q)$ of the structure sheaf Q of X is isomorphic to the Cohomology gorup $H^o(X,A)$ of the structure restricted sheaf A of X. Moreover, the Cohomology group $H^p(X,Q)$ of the structure sheaf Q of X and the Čech Cohomology group $H^p(\mathcal{U}, Q)$ of the structure sheaf Q of $\mathcal{U}$ equal to zero, for $p \geq 1$.

1- INTRODUCTION

Let X be a connected complex manifold of dimension n with fundamental group $H_x \neq \{1\}$, for any $x \in X$. Let $H = \underset{x \in X}{V} H_x$. A natural topology introduced on H in [1]. H is a sheaf with the cannocial projection mapping $\varphi : H \to X$ defined by $\varphi(\sigma_x) = x$, for every $\sigma_x \in H$. H is called the sheaf of the fundamental group. Let $\Gamma(X,H)$ be the group of global sections of X and $D \subset \Gamma(X,H)$ be the commutator subgroup. The subsheaf defined by D is called Commutator subsheaf of H and it is denoted by [H,H]. The Commutator subsheaf [H,H] is a normal subsheaf of H. The quotient sheaf $Q_{[H,H]}$ (or only Q) determined by [H,H] is a sheaf of Abelian groups and it is a regular covering space of X. The sheaf Q is isomorphic to the sheaf $\overline{H}$ of homology groups of X [1]. Hence, we identify the stalk Q_x with the stalk $\overline{H}_x$, for any $x \in X$, and the section $\gamma\ [s] \in \Gamma(X,Q)$ with the section $\bar{s} \in \Gamma\ (X,\overline{H})$.

We now give the following definition.

Definition 1.1. Let $(G_i)_{i \in IN}$ be a family of Abelian groups. Then,

i) A cochain complex is a sequence of group homomorphisms

$$G^* : G_0 \xrightarrow{d^0} G_1 \xrightarrow{d^1} G_2 \xrightarrow{d^2} \dots$$

with $d^i \circ d^{i-1} = 0$, for $i \in \mathbb{N}$.

ii) $Z^p(G^*) = \operatorname{Ker} d^p$ is called the p-th group of cocycles.

iii) $B^p(G^*) = \operatorname{Im} d^{p-1}$ is called the p-th group of coboundaries. We set $B^0(G^*) = 0$. Then clearly $B^p(G^*) \subset Z^p(G^*)$.

iv) The quotient group $H^p(G^*) = Z^p(G^*)/B^p(G^*)$ is called the p-th cohomology group of the complex G^*.

Finally, the homomorphism $d = d^p$ with $d^p \circ d^{p-1} = 0$ is called the coboundary operator, for $p \geq 0$.

Definition 1.2. An augmented cochain complex is a triple (E, ε, G^*) with the following properties:

i) E is an Abelian group.

ii) G^* is a cochain complex.

iii) $\varepsilon : E \to G_0$ is a monomorphism with $\operatorname{Im} \varepsilon = \operatorname{Ker} d^0$.

If (E, ε, G^*) is an augmented complex, then

$$E \cong \operatorname{Im} \varepsilon = \operatorname{Ker} d^0 = Z^0(G^*) \cong H^0(G^*).$$

From now on, X will be considered as a connected complex manifold of dimension n with fundamental group $H_x \neq \{1\}$, for any $x \in X$.

2. ČECH COHOMOLOGY GROUPS.

Let $\mathcal{U} = (U_i)_{i \in I}$ be an open covering of X with $U_i \neq \varnothing$ for every $i \in I$. It is shown, in this section, that;

i) The o-th Čech Cohomology group of U with values in Q is isomorphic to the Homology group $\overline{H}_x$ of X, for any $x \in X$.

ii) The p-th Čech Cohomology group $H^p(\mathcal{U}, Q)$ of $\mathcal{U}$ with values in Q equals to 0, for $p \geqslant 1$.

Let Q be the sheaf of Abelian groups determined by [H,H] over X and $\mathcal{U} = (U_i)_{i \in I}$ be an open covering of X with $U_i \neq \varnothing$ for every $i \in I$. We define,

$$U_{i_0 \dots i_p} = U_{i_0} \cap \dots \cap U_{i_p}$$

$$I_p = \{(i_0, \dots, i_p) : U_{i_0 \dots i_p} \neq \varnothing\}.$$

Let τ_n be the set of permutation of the set $\{0,1,2,\ldots,(n-1)\}$. For $\tau \in \tau_n$, let

$$\operatorname{sgn}(\tau) = \begin{cases} +1, & \text{if } \tau \text{ is the product of an even number of transpozitions} \\ -1, & \text{otherwise} \end{cases}$$

Definition 2.1. An p-dimensional (alternating) cochain over $\mathcal{U}$ with values in Q is a mapping

$$m: I_p \to \bigcup_{(i_0,\ldots,i_p)} \Gamma(U_{i_0}\ldots i_p, Q)$$

with the following properties:

i) $m(i_0,\ldots,i_p) \in \Gamma(U_{i_0}\ldots i_{i_p}, Q)$

ii) $m(i\tau(0),\ldots,i\tau(p)) = \operatorname{sgn}(\tau).\ m(j_0,\ldots,i_p)$, for $\tau \in \tau_{p+1}$

The set of all p-dimensional alternating cochains over U with values in Q denoted by $C^p(\mathcal{U}, Q)$. $C^p(\mathcal{U}, Q)$ becomes an Abelian group by setting

$$(m_1 + m_2)(i_0,\ldots,i_p) = m_1(i_0,\ldots,i_p) + m_2(i_0,\ldots,i_p).$$

Let us now define a mapping,

$d = d^p : C^p(\mathcal{U}, Q) \to C^{p+1}(\mathcal{U}, Q)$ with

$$(dm)(i_0,\ldots,i_{p+1}) = \sum_{\lambda=0}^{p+1} (-1)^{\lambda+1}(m(i_0,\ldots,\hat{i}_\lambda,\ldots,i_{p+1})\,|U\, i_0\ldots i_{p+1}),$$

where $\hat{i}_\lambda$ means that the index i_λ is delated.

It is easily seen that d is a homomorphism with $d^{p+1} o d^p = 0$.

Definition 2.2. The sequence

$$C^*(\mathcal{U},Q) : C^0(\mathcal{U},Q) \xrightarrow{d^0} C^1(\mathcal{U},Q) \xrightarrow{d^1} C^2(\mathcal{U},Q) \xrightarrow{d^2} \cdots$$

is called the čech complex.

Let us now define a mapping $\varepsilon : \Gamma(X,Q) \to C^0(\mathcal{U},Q)$ with $(\varepsilon\bar{s})(i) = \bar{s}\,|U_i$, for every $\bar{s} \in \Gamma(X,Q)$. Then we can give,

Theorem 2.1. The triple $(\Gamma(X,Q), \varepsilon, C^*(\mathcal{U},Q)$, is an auqmented cochain complex.

Proof. Clearly, ε is a homomorphism. If $\varepsilon\, \bar{s} = o$, then $\bar{s}\ |u_i = 0$, for every $i \in I$; therefore $\bar{s} = 0$. Hence ε is injective.

Let $m \in C^o(\mathcal{U},Q)$ and dm=o. Since,

$$(dm)\ (i_o,i_1) = (-m(i_1) + m(i_o)) \mid U_{i_0 i_1}$$

this is equivalent to $m(i_o) \mid U_{i_0 i_1} = m(i_1) \mid U_{i_0 i_1}$. Therefore there is a section $\bar{s} \in \Gamma\,(X\ Q)$ with $\varepsilon\, \bar{s} = m$ defined by $\bar{s} \mid U_i = m(i)$. Thus, $\mathrm{Im}\ \varepsilon = \mathrm{Ker}\ d^o$.

Definition 2.3. Let $C^*(\mathcal{U},Q)$ be the Čech complex and p ($\geqslant 0$) be an integer.

i) $Z^p(\mathcal{U},Q) = \mathrm{Ker}\ d^p$ is called the group of p-th cocycles over $\mathcal{U}$ with values in Q

ii) $B^p(\mathcal{U},Q) = \mathrm{Im}(d^{p-1})$ is called the group of p-th coboundaries over $\mathcal{U}$ with values in Q.

Clearly, $B^p(\mathcal{U},Q) \subset Z^p(\mathcal{U},Q) \subset C^p(\mathcal{U},Q)$.

iii) The quotient group $H^p(\mathcal{U},Q) = Z^p(\mathcal{U},Q)\,/B^p(\mathcal{U},Q)$ is called the p-th Čech Cohomology group of U with values in Q [2,3].

In particular, $H^o(\mathcal{U},Q) \cong \Gamma(X,Q)$. On the other hand, $\Gamma(X,Q) \cong \Gamma(X,\bar{\bar{H}}) = \bar{\bar{H}}_X$. Therefore, $H^o(X,Q) \cong \bar{\bar{H}}_X$, i.e., the o-th Čech Cohomology group of U with values in Q is isomorphic to the Homology group of X, for any $x \in X$.

Definition 2.3. Let S be a sheaf over X. If the restriction mapping $\gamma_{X,U}: \Gamma(X,S) \to \Gamma(U,S)$ is a surjection for any open set $U \subset X$, then S is called a flabby sheaf.

It is easy to see that the sheaves H, [H,H] and Q are flabby sheaves by considering their constructions, respectively.

Let O be zero sheaf or identy sheaf. The sequence,

$O \to [H,H] \xrightarrow{i} H \xrightarrow{\pi} Q \to O$ is exact, where the mapping i is cannonical injection and the mapping π is cannonical surjection. Let $\gamma\,[s] \in \Gamma(X,Q)$. Then, there exists a unique element $[s] \in \Gamma(X,H)\,/\Gamma(X,[H,H])$ such that $\gamma\ [s] = \bar{s}$, by means of the isomorphism between Q and H. So, there is at least one section $s \in \Gamma(X,H)$ such that $\gamma\,[s] \in \Gamma(X,Q)$. Since the mapping $\pi : H \to Q$ is cannonical projection, $(\pi\ os)\ (x) = \gamma\ [s]\ (x)$, for every $x \in X$. Then we may state, *Theorem* 2.2. The Sequence,

$$0 \to \Gamma(X, [H,H]) \overset{i_*}{\to} \Gamma(X,H) \overset{\pi_*}{\to} \Gamma(X,Q) \to 0$$

is exact.

Theorem 2.3. Let Q be the sheaf of Abelian groups determined by [H,H] over X, $\mathcal{U} = (U_i)_{i \in I}$ be an open covering of X with $U_i \neq \varnothing$ and $X \in \mathcal{U}$. Then, $H^p(\mathcal{U},Q) = 0$, for $p \gg 1$.

Proof. If $\mathcal{U} = (U_i)_{i \in I}$, then there is an $r \in I$ with $X = U_r$. Let $m \in Z^p(\mathcal{U},Q)$, $p \geq 1$. There is an element $n \in C^{p-1}(\mathcal{U},Q)$ defined by $n(i_0,\ldots,i_{p-1}) = m(r,i_0,\ldots,i_{p-1})$. Since $dm = 0$, we have

$$0 = dm\ (r,i_0,\ldots,i_p) = -m(i_0,\ldots,i_p) + \sum_{\lambda=0}^{p} (-1)^{\lambda}\, m(r,i_0,\ldots,\hat{i}_\lambda,\ldots,i_p)$$

Therefore,

$$d(-n)\ (i_0,\ldots,i_p) = -\sum_{\lambda=0}^{p} (-1)^{\lambda+1}\, n(i_0,\ldots,\hat{i}_\lambda,\ldots,i_p)$$

$$= \sum_{\lambda=0}^{p} (-1)^{\lambda}\, m(r,i_0,\ldots,\hat{i}_\lambda,\ldots,i_p) = m(i_0,\ldots,i_p).$$

In other words, $d(-n) = m$, so $m \in B_p(\mathcal{U},Q)$. Namely, the Čech Cohomology sequence is exact at every location $p \geq 1$, i.e., $H^p(\mathcal{U}, Q) = 0$, for $p \geq 1$.

We now give the following theorem.

Theorem 2.4. Let $\mathcal{U}$ be an arbitrary covering of X. Then, $H^p(\mathcal{U},Q) = 0$, for $p \geq 1$.

Proof. We prove this tehorem by induction on p. Let $p \geq 1$ and $m \in Z^p(\mathcal{U},Q)$. If $U \subset X$ is an open set, then we set $U \cap \mathcal{U} = \{U \cap U_i \neq \varnothing : U_i \in \mathcal{U}\}$ and

$$(m\,|U)\ (i_0,\ldots,i_p) = m(i_0,\ldots,i_p)\,|U \cap U_{i_0} \ldots {i_p}.$$

With this notation we have $m\,|U \in Z^p(U \cap \mathcal{U},Q)$.

For arbitrary $x_0 \in X$, there is an $i_0 \in I$ and an open neighborhood $U(x_0) \subset U_{i_0}$. But then $U \in U \cap \mathcal{U}$, so $H^p(U \cap \mathcal{U},Q) = 0$, for $p \geq 1$, and there is an $n \in C^{p-1}(U \cap \mathcal{U},Q)$ with $dn = m\,|U$.

If $V \subset X$ is an open set with the same property, i.e., there is an $n' \in C^{p-1}(V \cap \mathcal{U},Q)$ with $n' = m|V$, we set

$$t = (n-n')|U \cap V \in Z^{p-1}((U \cap V) \cap \mathcal{U},Q).$$

If $p=1$, then t lies in $\Gamma(U \cap V,Q)$, and since Q is flabby, we can extend t to a $\hat{t} \in \Gamma(V,Q)$. Then set

$$t^*(x) = \begin{cases} n(x), & x \in U \\ n'(x) + \hat{s}(x) & x \in V. \end{cases}$$

Clearly $t^* \in \Gamma(U \cup V,Q)$ and $dt^* = m|U \cup V$, because $d\hat{s} = 0$.

If $p > 1$, then by the induction hypothesis there is a $\gamma \in C^{p-2}(U \cap V \cap \mathcal{U},Q)$ with $d\gamma = t$. Since Q is flabby,

$$\gamma(i_0,\ldots,i_{p-2}) \in \Gamma(U \cap V \cap U_{i_0 \ldots i_{p-2}},Q)$$

can be extended to an element

$$\hat{\gamma}(i_0,\ldots,i_{p-2}) \in \Gamma(V \cap U_{i_0 \ldots i_{p-2}},Q).$$

Let

$$n^*(i_0,\ldots,i_{p-1})(x) = \begin{cases} n(i_0,\ldots,i_{p-1})(x) & \text{for } x \in U \cap U_{i_0 \ldots i_{p-1}} \\ (n'+d\hat{\gamma})(i_0,\ldots,i_{p-1})(x) & \text{for } x \in V \cap U_{i_0 \ldots i^{p-1}} \end{cases}$$

Then $n^* \in C^{p-1}((U \cup V) \cap \mathcal{U},Q)$ and $dn^* = m|U \cup V$.

By Zorn's lemma there must be a maximal element (U_o,t_o) for $p=1$, resp. (U_o,n_o) for $p > 1$ with $t_o \in \Gamma(U_o,Q)$ and $dt_o = m|U_o$, resp. $n_o \in C^p(\mathcal{U},Q)$ and $dn_o = m|U_o$. But an element is only maximal if $U_o = X$; therefore $m \in B^p(\mathcal{U},Q)$. Hence, $H^p(\mathcal{U},Q) = 0$.

3 FLABBY COHOMOLOGY GROUPS

In this section, it is shown that;

i) The o-th Cohomology group $H^o(X,Q)$ of X with values in Q is isomorphic to the Homology group $\overline{H}_x$ of X for any $x \in X$.

ii) The p-th Cohomology group $H^p(X,Q)$ of X with values in Q equals to zero for $p \geq 1$.

Let Q be the sheaf of Abelian groups determined by [H,H] over X and $U \subset X$ be an open set. Let $\hat{\Gamma}(U,Q)$ denote the set of all mappings $f: U \to Q$ with ψ o$f = 1_U$, where $\psi: Q \to X$ is the sheaf projection. We call these not necessarily continuous functions generalized sections. Clearly $\Gamma(U,Q)$ is a subgroup of $\hat{\Gamma}(U,Q)$. We set $M_U = \hat{\Gamma}(U,Q)$. If U, $V \subset X$ are open with $V \subset U$, then we define $\gamma_{U,V}: M_U \to M_V$ by $\gamma_{U,V}(f) = f|V$. Then $\{X, M_U, \gamma_{U,V}\}$ is a pre-sheaf and we denote the corresponding sheaf by W(Q) [2].

Theorem 3.1.

1. The cannonical mapping $\gamma : M_U \to \Gamma(U,W(Q)$ is a group homomorhism.

2. The cannonical injection i_U: $\Gamma(U,Q) \subset \hat{\Gamma}(U,Q)$ induces an injective sheaf homomorphism ε: $Q \to W(Q)$ with $\varepsilon_* \,|\Gamma(U,Q) = \gamma\, o i_U$, where γ is the inductive limit operator.

Proof. 1. A similar proof can be found for 1 in [1].

2. Clearly $i_U(\bar{s}) \mid V = i_V(\bar{s}) \mid V)$ for $\bar{s} \in \Gamma(U,Q)$. If we identify the sheaf induced by $\{X, M_U, \gamma_{U,V}\}$ with the sheaf Q, then there exists exactly one sheaf morphism ε: $Q \to W(Q)$ with $\varepsilon_*(\bar{s}) = \gamma o i_U(\bar{s})$ for $\bar{s} \in \Gamma(U,Q)$ [1]. If $\bar{\sigma} \in Q_x$ and $\varepsilon(\bar{\sigma}) = O_x$, then there exists a neighborhood $U(x) \subset X$ and an $\bar{s} \in \Gamma(U,Q)$ with $\bar{s}(x) = \bar{\sigma}$. Therefore, $O_x = \varepsilon(\bar{\sigma}) = \varepsilon o \bar{s}(x) = \varepsilon_*(\bar{s})\,(x) = \gamma_o i_U(\bar{s})\,(x)$ with $\gamma\, i_U(\bar{s}) \in \Gamma(U,W(Q))$. Then there exists a neighborhood $V(x) \subset U$ with $\gamma\, i_U(\bar{s}) \mid V = O$; thus $i_U(\bar{s}) \mid V = O$ and then clearly $\bar{s} \mid V = O$. Hence $\bar{\sigma} = \bar{s}(x) = O_x$.

Let $W_o(Q) = W(Q)$. Let us construct the sequence

$$O \to Q \xrightarrow{\varepsilon} W_o(Q) \xrightarrow{d^o} \dots \xrightarrow{d^{p-1}} W_p(Q) \dots$$

Where $I_m(d^{-1}) = \mathrm{Im}\,\varepsilon$, $W_{p+1} = W(W_p(Q)/\mathrm{Im}(d^{p-1})$ and $d = d^p = j \circ q$ for the cannonical projection q: $W_p(Q) \to W_p(Q)/I_m(d^{p-1})$ and the cannonical injection j: $W_p(Q)/I_m(d^{p-1}) \to W(W_p(Q)/I_m(d^{p-1}))$. Clearly $\mathrm{Ker}\, d^p = \mathrm{Ker}\, q = \mathrm{Im}(d^{p-1})$. Thus the sequence

$$O \to Q \xrightarrow{\varepsilon} W_o(Q) \xrightarrow{d^o} \dots \xrightarrow{d^{p-1}} W_p(Q) \dots$$

is exact and it is called the cannonical resolution of Q.

Theorem 3.2. Let Q be the sheaf of Abelian groups determined by [H,H] over X and $W^*(Q)$: $\Gamma(X,W_o(Q)) \to \Gamma(X,W_1(Q)) \to \Gamma(X,W_2(Q)) \dots$ Then the triple $(\Gamma(X,Q), \varepsilon_*, W^*(Q))$ is an augmented cochain complex.

Proof. Clearly $W^*(Q)$ is a cochain complex. The mapping $\varepsilon_*: \Gamma(X, Q) \rightarrow \Gamma(X,W_0(Q))$ is a group homomorphism and $(d^o)_* \circ \varepsilon_* = O$.

Consider the mapping

$$d^o: W_0(Q) \overset{q}{\rightarrow} W_0(Q)/I_{m\varepsilon} \subset W(W_0(Q)/_{Im\varepsilon}) = W_1(Q).$$

Let $f \in \Gamma(X,W_0(Q))$ and $O = d^o \circ f =$ joqof. Then qof $= O$, so $f(x) \in$ Im ε for every $x \in X$. Since Im $\varepsilon \cong Q$, $\Gamma(X$, Im $\varepsilon) \cong \Gamma(X,Q)$. Thus there is an element $\bar{s}^* \in \Gamma(X,Q)$ such that $\varepsilon_*(\bar{s}^*) = \bar{s}$.

Definition 3.1. Let Q be the sheaf of the Abelian groups determined by [H,H] over X and $(\Gamma(X,Q), \varepsilon_*, W^*(Q))$ be the augmented cochain compleks.

i) $Z^p(X,Q) =$ Ker d^p is called the group of p-th cocycles of X with values in Q.

ii) $Z^p(X,Q) =$ Im(d^{p-1}) is called the group of p-th coboundaries of X with values in Q.

iii) The quotient group $H^p(X,Q) = Z^p(X,Q)/B^p(X,Q)$ is called p-th cohomology group of X with values in Q.

In particular, $H^o(X,Q) \cong \Gamma(X,Q)$. On the other hand, $\Gamma(X,Q) \cong \overline{H}_x$. Therefore, $H^o(X,Q) \cong \overline{H}_x$, i.e., o-th cohomolony group $H^o(X,Q)$ of X with values in Q is isomorphic to the homology group $\overline{H}_x$ of X for any $x \in X$.

Let us now consider the sequence

$0 \rightarrow \Gamma(X,Q) \rightarrow \Gamma(X,W_0(Q)) \rightarrow \Gamma(X,W_1(Q) \rightarrow \ldots.$ For $p = 0, 1, 2,\ldots$, let $B_p =$ Im (d^{p-1}) and $d^{-1} = \varepsilon$. By the induction, it is shown that all B_p are flabby. In fact, for $B_0 = Q$ this is true. Suppose that $B_0,B_1,\ldots,B_{p-1}$ are flabby sheaves. Since the sequence $O \rightarrow B_{p-1} \rightarrow W_{p-1}(Q) \rightarrow W_p(Q) \rightarrow O$ is exact, the sequence $0 \rightarrow \Gamma(U,B_{p-1}) \rightarrow \Gamma(U, W_{p-1}(Q)) \rightarrow \Gamma(U,W_p(Q)) \rightarrow 0$ is exact for the open $U \subset X$. Let $f \in \Gamma(U,B_p)$. Then there exists a section $f' \in \Gamma(U,W_{p-1}(Q))$ such that $d^p o\, f = f'$. Since the sheaf $W_{p-1}(Q)$ is flabby there exists a section $f^* \in \Gamma(X,W_{p-1}(Q))$ with $f^* \mid U = f'$. But $d^p \circ f^* \in \Gamma(X,W_p(Q))$ and d^p of $^* \mid U = f$. Therefore, B_p is flabby.

On the other hand, the following sequences

$$O \rightarrow B_{p-1} \rightarrow W_{p-1}(Q) \rightarrow B_p \rightarrow O$$

$$0 \to B_p \to W_p(Q) \to B_{p+1} \to 0$$
$$0 \to B_{p+1} \to W_{p+1}(Q) \to B_{p+2} \to 0$$

are exact. Thus the corresponding sequences of groups of sections are exact [2]. Therefore the sequence

$$\Gamma(X,W_{p-1}(Q)) \to \Gamma(X,W_p(Q)) \to \Gamma(X,W_{p+1}(Q))$$

is exact. Then we may state,

Theorem 3.3. Let Q be the sheaf of Abelian groups determined by [H,H] over X. Then $H^p(X,Q) = 0$ for $p \geq 1$.

4. THE MAIN RESULT

Let X be a connected complex analytic manifold with fundamental group $H_x \neq \{1\}$, for any $x \in X$ and A(X) be the vector space of all holomorphic functions of X. Let $f \in A(X)$ and $x \in X$ a point. f can be expanded into a power series f_x convergent at z the local parameter of x. The totality of such power series at x as f runs through A(X) is denoted by A_x which is again a vector space (or C -Algebra) isomorphic to A(X). The disjoint uniou $A = \bigvee_{x \varepsilon X} A_x$ is a set over X with a natural projection $\pi : A \to X$ mapping each f_x onto the point of expansion x.

A natural topology on A was introduced in [4]. In that topology π is locally topological mapping. Hence (A, π) is a sheaf over X. The sheaf A is called the Restricted Sheaf of germs of the totality of holomorphic functions A(X) on X [4]. In paper [4] it is shown that the cohomology group $H^o(X,A)$ of X with values in A is isomorphic to the homology group $\overline{H}_x$ of X. In this paper, we show that $\overline{H}_x$, is isomorphic to the cohomology group $H^o(X,Q)$ $(= H^o(\mathcal{U},Q))$. Then,

i) $H^o(X,Q) \cong H^o(\mathcal{U},Q) \cong H^o(X,A) \cong H^o(\mathcal{U},A)$.

ii) $H^p(X,Q) = H^p(\mathcal{U},Q) = 0$, for $p \geq 1$.

Let $Q' \subset Q$ be a subsheaf. Then there corresponds a subsheaf $A' \subset A$ to Q'. It can be shown, in similar way, that

$$H^o(X,Q') \cong H^o(\mathcal{U},Q') \cong \Gamma(X,Q') \subset \Gamma(X,Q).$$

Since Q' is flabby

i) $H^o(X,Q') \cong H^o(\mathcal{U},Q') \cong H^o(X,A') \cong H^o(\mathcal{U},A')$.

ii) $H^p(X,Q') \cong H^p(\mathcal{U},Q') = 0$ for $p \geq 1$.

REFERENCES

[1] BALCI, S., On the Restricted Sheaf, Comm. Fac. Sci. Univ. Ankara, Ser. A_1: Mathematiques and Statistics (accepted for pubication).

[2] GRAUERT, H., FRITSCHE, K., Several Complex Variables, Springer-Verlag (1976).

[3] SERRE, J.-P., FASIC. Eaux Algebraiques Cohéerents, Ann. Math. Vol. 61., pp.197–278 (1955).

[4] ULUÇAY, C., On the Homology group of the Complex Analytic Manifolds, Comm. Fac. Sci. Univ. Ankara, Ser A_1: Mathematiques, Tome: 30, pp. 37–44 (1981).