COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A_1: Mathématiques

TOME 27 ANNÉE 1978

Optical Fragments Of The Real Affine Plane

Rüstem KAYA

2

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les disciplines scientifique représentées à la Faculté.

La Revue, Jusqu'à 1975 à l'exception des tomes I, II, III, etait composé de trois séries

Série A : Mathématiques, Physique et Astronomie.
Série B : Chimie.
Série C : Sciences naturelles.

A partir de 1975 la Revue comprend sept séries:

Série A_1: Mathématique
Série A_2: Physique
Série A_3: Astronomie
Série B : Chimie
Série C_1: Géologie
Série C_2: Botanique
Série C_3: Zoologie

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté. Elle accepte cependant, dans la mesure de la place disponible, les communications des auteurs étrangers. Les langues allemande, anglaise et française sont admises indifféremment. Les articles devront être accompagnés d'un bref sommaire en langue turque.

Adresse: Fen Fakültesi Tebliğler Dergisi, Fen Fakültesi, Ankara, Turquie.

Optical Fragments Of The Real Affine Plane

Rüstem KAYA

Fen Fakültesi Matematik Bölümü ANKARA

(Received 5 October 1978, and accepted 31 October 1978)

ABSTRACT

In this paper by deleting certain lines of the real affine plane and adding some new broken lines to it the concept of optical fragments of real affine plane is defined. Then it is shown by an example that such a fragment corresponds to the motion of light rays in a two dimentional space which consists of two distinct mediums separated along a straight line from each other.

Menger [3] defined the self-dual fragments of the real affine plane by deleting all the lines parallel to the y-axis or an ordinary point P and all the lines through P. In a self-dual fragment point and lines play precisely equal roles in contrast to the affine plane itself. There is a complete analogue between the self-dual fragment of the affine plane and Galilean kinematics of the uniform motions in a one-dimentional space embedded in a two dimensional space-time world. Briefly, the self-dual fragment of the affine plane represents the Galilean kinematics of the uniform motions in a one-dimensional space (see Blumental-Menger [1, §. 5.4]). Very recently, Seall [5] used the self-dual fragments to develope projective planes ovei superasociative systems. The concept of affine planes with broken lines has been introduced by Moulton [4] to show that Desargues' theorem is not a consequence of certain axioms of Hilbert [2] for the plane geometry.

In this note using the idea due to Menger and Moulton some new fragments of the real affine plane are defined for which following postulates are valid:

F1. If P and Q are two distinct points, then there exists at most one line that joins P and Q.

F2. If l and m are two distinct lines, then there exists at most one point that is on both l and m.

F3. If l is a line and P a point not on l, then there exists a unique line that is on P and parallel to l.

Obviously, F2 and F3 are the axioms of an affine plane while F1 is weaker than than the corresponding postulate for an affine plane.

Definition: Let A denote the real affine plane and let k be a given real niumber with $k > 1$. Let us delete all the half-lines with the slopes in the intervals $(0, +\sqrt{k^2 - 1}]$ and $[-\sqrt{k^2-1}, 0)$ from the lower half part of A, but retain the points on the deleted half-lines. Where with lower part of A we mean the part where $y < 0$. Let new lines of the deleted A have equations as follows:

$$y = \begin{cases} m(x - n) & \text{if } m > 0 \text{ and } y \geqslant 0 \\ +\sqrt{k^2(m^2+1) - 1}\,(x - n) & \text{if } m > 0 \text{ and } y < 0 \end{cases}$$

$$y = \begin{cases} m(x - n) & \text{if } m < 0 \text{ and } y \geqslant 0 \\ -\sqrt{k^2(m^2+1) - 1}\,(x - n) & \text{if } m < 0 \text{ and } y < 0 \end{cases}$$

and $x = a$ if $m \to \infty$; $y = b$ if $m = 0$. The collection of points and lines thus obtained will be called an *optical fragment of* A and denoted by F.

Proposition: The optical fragment F satisfies F1, F2 andF3.

Two distinct lines in F are said to be parallel if and only if they have no point in common. Thus the refracted lines are parallel if only if they are parallel in the upper half plane in the euclidean sense. Every line has a continuation in the lower part of F. Since for given m_1 and m_2, $m_1 \neq m_2$, only one of the points

$$((m_1 n_1 - m_2 n_2)(m_1 - m_2)^{-1},\ m_1 m_2 (n_1 - n_2)(n_1 - n_2)(m_1 - m_2)^{-1})$$

and

$$([n_1 S(m_1) - n_2 S(m_2)]\,[S(m_1) - S(m_2)]^{-1},$$

$$[S(m_1) S(m_2)(n_1 - n_2)]\,[S(m_1) - S(m_2)]^{-1})$$ is meaningful, any two lines with distinct slopes meet or do not meet in the upper half of F if and only if refracted parts of them do not meet or meet in the lower half of F respectively. Where $S(m_i)$ denotes the cor-

responding slope of the continuatuion of the line with the slope m_i. Thus, F2 and F3 are satisfied.

For any two points in the upper half of F there exists exactly one line joining them; while any two points given in the lower half of F, say (x_1, y_1) and (x_2, y_2) with $y_1, y_2 < 0$, $x_1 \neq x_2$ and $y_1 \neq y_2$, there exists no line if $0 < (y_2 - y_1)(x_2 - x_1)^{-1} < +\sqrt{k^2-1}$ or $-\sqrt{k^2-1} < (y_2 - y_1)(x_2 - x_1)^{-1} < 0$, and a unique line if this is not the case, that is, if $(y_2 - y_1)(x_2 - v_1)^{-1} > \sqrt{k^2 - 1}$ or

$(y_2 - y_1)(x_2 - x_1)^{-1} < -\sqrt{k^2-1}$. Let us consider the case where $x_1 \neq x_2$, $y_1 > 0$, $y_2 < 0$ and $(y_2 - y_1)(x_2 - x_1)^{-1} > 0$. Hence m is to be positive and satisfies the equation

$$+\sqrt{k^2(m^2+1)-1}\,[(x_2 - x_1)m + y_1] - my_2 = 0 \qquad (1)$$

A positive root of Equation (1) is necessarily between

$-y_1(x_2 - x_1)^{-1}$ and $(y_2 - y_1)(x_2 - x_1)^{-1}$. Let

$f(m) = +\sqrt{k^2(m^2+1)-1}\,[(x_2 - x_1)m + y_1] - my_2$.

Since $f(-y_1(x_2-x_1)^{-1}) > 0$ and $f((y_2 - y_1)(x_2 - x_1)^{-1}) < 0$ there is at least one root of Equation (1) between $-y_2(x_2 - x_1)^{-1}$ and $(y_2-y_1)(x_2-x_1)^{-1}$. If there were more than one positive root of Equation (1), in the above interval, two distinct lines would have met on two distinct points which is contrary to F2. Hence there is a unique line that is on both (x_1, y_1) and (x_2, y_2). A similar proof can be given for the case where $(y_2 - y_1)(x_2 - x_1)^{-1} < 0$. The other cases are obvious and thus F1 is satisfied.

An Example For The Optical Fragment

Let us consider a two dimensional space which consists of two distinct mediums separated along a straight line from each other. In such a space the paths of the light rays moving from the first medium into the second, such that refractive index of the first medium is less than that of the second medium, can be taken as lines of a fragment F of the plane. This fact is obvious from the well known Snell Law: *For the light incident on a boundary between two media, the ratio of sine of the angle of incidence (the angle between the light ray in the first medium and the normal to the boundary* *surface) to the sine of the angle of the refraction (the angle between the refracted ray in tde second medium anh the normal) is a constant, being equal to the inverse ratio of the refractive indices of the two media.* For instance, if the first medium is air and the second medium water both having the temperature 20 °C for the sodium light with the wave lenght 5893 Angström the ratio of the feractive indices is 1.33299. By taking $k = 1.33299$ and using the Snell Law it can be easily calculated that no refracted light ray makes an angle with the boundary line in the intervals $(\pi, \pi + \alpha)$ and $(2\pi - \alpha, 2\pi)$, where $\cos \alpha = k^{-1}$, $0 < \alpha < \pi/2$, and therefore $\alpha \cong 41° 24'$. Clearly, the x – axis of the fragment, that is, the line which separates the two mediums is an exception and it is defined especially as $y = 0$ for the fragment.

An optical fragment of three dimensional real affine space (can be obtained by deleting certain half planes and certain half lines from the space and defining new planes and lines in a similar way to that of two dimensional case. Beside this, it is also possible to define the so called optical fragments for $0 < k < 1$.

REFERENCE

[1] **Blumental, L.M.-Menger. K.:** Studies in Geometry, W.H. Freeman and Comp., San Francisco, (1970).

[2] **Hilbert. D.:** Foundations of Geometry, The Open Court Publ. Comp. La Salle, Illinois, (1950).

[3] **Menger. K.:** Self-Dual Fragments Of The Plane, Amer. Math. Monthly 56, pp. 545–546, (1949).

[4] **Moulton. S.R.:** A Simple Non-Desarguesian Plane Geomeryt, Trans. Amer. Math. Soc. 3, pp. 192–195, (1902).

[5] **Seall. R.:** Connections Between Projective Geometry and Super-associative Algebra, Journal of Geometry, 4, 1, (1974).

ÖZET

Bu makalede gerçel afin düzlemden bir takım doğruların çıkarılması ve ona kırık çizgilerden meydana gelen bazı yeni doğruların katılmasıyla gerçel afin düzlemin optiksel parçaları kavramı tanımlanmaktadır. Daha sonra bunların bir doğru boyunca ayrılan farklı iki ortamdan oluşan düzlemsel bir alanda ışınların hareketini temsil ettikleri örnekle gösterilmektedir. Dolayısıyla, böyle bir ortamda ışık ışınlarının geometrisi belirlenmektedir.

Prix de l'abonnement annuel

Turquie : 15 TL; Étranger: 30 TL.
Prix de ce numéro : 5 TL (pour la vente en Turquie).
Prière de s'adresser pour l'abonnement à : Fen Fakültesi Dekanlığı Ankara, Turquie.

Ankara Üniversitesi Basımevi. Ankara - 1979