Note on the zero point correction of Galactic Cepheids

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Özet : σ_p, gerçel hızların dağılımındaki radyal ve teğetsel doğrultularda standart sapma olarak, $\partial\sigma_p/\partial V_{\odot}$ ve $\partial\sigma_p/\partial a$ şartları, benzer tarzda teşkil edilmesi lazımgelen ve $V_{\odot}$ ve absorpsiyon miktarlarını ihtiva eden iki denklem çıkarmak için kullanılmışlardır. Eğer period, mutlak parlaklık bağıntısının sıfır noktasının tashihi için bir f çarpanı hesaba dahil edilirse, $V_{\odot}$, f ve $a(y)$ bilinmeyen kemiyetleri ihtiva eden iki seri denklem elde edilir. Bu kemiyetlerden biri belli olduğu zaman diğer ikisi hesaplanabilir. Denklemler, radyal hızları ve öz hareketleri bilinen parlak galaktik cepheidlere tatbik edilmiştir. Yalnız sıfır noktası tashihi maksimum değeri $\Delta M = -2.40$ ve minimum değeri $a(y) = -0.000.6$ Mag/parsek elde edilmiştir.

Her iki (ΔM) ve $a(y)$ nümerik değerlerinin kullanılan y mesafe parametresinin sınırlanan değerine tabi olduğu görülür. Bu bağımlılık gerçel değildir, fakat neticeler yıldızların seçilme tarzına bağlıdır.

Bu makalede geliştirilen metod hali hazırda yok edilmesi güç olan sistematik tesirlere epeyce hassastır.

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Abstract : σ_p being the standard deviation in the distribution of the peculier velocities in the radial and tangential directions, the conditions $\partial\sigma_p/\partial V_{\odot}$ and $\partial\sigma_p/\partial a$ are used to derive two equations whicht must simultaneously be fulfilled and which contain as unknown quantities $V_{\odot}$ and the absorption. If a factor f is introduced to account for the correction to the zero point of the period absolute magnitude relation, two sets of equations are obtained which contain the unknown quantities $V_{\odot}$, f and $a(y)$. When one of these quantities is known the other two can be computed. The equations have been applied to the bright galactic Cepheids for which the radial velocities and proper motions are known. Only a maximum value of the zero

point correction $\Delta M = -2.40$ and a minimum value of $a(y) = -0.00016$ mag/parsecs can be obtained.

It appears that the numerical values of both f (and ΔM) and $a(y)$ depend on the limiting value of the distance parameter y which is used. This dependence is not real but results from the way in which the stars have been selected.

The method developed in the present paper is rather sensitive to such systematic effects, which at present are difficult to eliminate.

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§ 1. Introduction.

By his research on the Audromeda nebulae Baade [1] proved convincingly that with the Cepheids a correction of about - 1.4 magnitudes should be applied to the zero point of the period - absolute magnitude relation. This was supported by evidence obtained by Thackeray and Wesselink [2] from their work on the cluster type variables in the globular clusters connected with the Magellanic clouds.

Later on Blaauw and Morgan [3] have determined the amount of correction from the secular parallaxes of galactic Cepheids. Their results are based on the 18 stars for which the proper motions can be most accurately computed at the present. For the numerical amount of the correction to be applied to Shapley's curve the value $\Delta M = -1^m.4 \pm 0^m.3$ $(p:e)$ is obtained while the correction to be applied to the the analytical curve of Paranago is found to be $\Delta M = -1^m.44$. In their calculations for the fotographic absorption a value of $0^m.7$ per thousand parsecs was adopted. From a foto-electric fotometry of galactic Cepheids Eggen, Gascoigne and Burr [4] conclude that the magnitudes used by Blaauw and Morgan require a correction of $-0^m.17$. In a subsequent paper Gascoigne and Eggen [5] find that a total correction of $-1.^m7$ is to be applied to the zero point. They obtain the corrections for absorption from the observed colour excess. All their magnitudes are foto-electrically determined. In a previous paper Eggen [6] has indicated that little reliance could be placed on much of the fotographic data except for a study of the periods. In the present paper an attemp is made directly to derive both the values of the interstellar absorption and the correction to the zero point from the observed radial velocities and proper motions. For this a method is used which is briefly outlined in § 2. Just as in the papers mentioned above, in the application of the method it was assumed that the absolute magnitude of a Cepheid is uniquely determined by its period. This assumption is generally known not to be quite correct. There is a certain scattering of the individual magnitudes around a mean value. This scattering certainly is not negligible, but is difficult to account for being imperfectly known. However, as will appear from our results, to a large extent the uncertainties in the determination of the zero point and of the fotographic absorption are due to this scatter of the individual magnitudes. Also when the sample of Cepheids used in the determination is not statistically unbiased this scatter of the individual magnitudes will cause a systematic error in the determination of the zero point correction, the amount of this correction being overestimated.

In our results the influence of these systematic errors clearly show up. This is hardly astonishing as the samples which can be used are known to be biased. As will appear from the following, the method used in the present paper is rather sensitive to these effects and our results therefore may be more strongly affected than those of the other authors.

§ 2. Outline of the method

Let $r = y \, exp\,(y\, a(y))$ where r is the true distance of the star and y the apparent distance e. g. the distance which is obtained from the apparent magnitude when the interstellar absorption is entirely neglected. For small and intermediate distances we write :

$$r = y\{1 + y\, a(y)\} \qquad \cdots (1)$$

The peculiar velocities in radial direction and in the directions α and δ are :

$$\left.\begin{aligned} V_{pr} &= V_r - V_{\odot} \cdot \lambda_1 - y\, R(l, b) - a y^2\, R(l, b) \\ V_{p\alpha} &= k\,(\mu_\alpha\, y + V'_{\odot}\, \lambda_2 + y^2\, a\, \mu_\alpha) \\ V_{p\delta} &= k\,(\mu_\delta\, y + V'_{\odot}\, \lambda_3 + y^2\, a\, \mu_\delta) \end{aligned}\right\} \qquad \cdots (2)$$

where R represents the term for differential galactic rotation

$$R = A \sin (l - l_0) \cos^2 b \qquad \text{ve} \qquad V'_\odot = V_\odot / k = V_\odot / 4.74$$

while λ_1, λ_2 and λ_3 are the directional cosines of the solar velocity. μ_α and μ_δ are the values of the proper motion after elimination of the effects of precession and differential galactic rotation. Now let $\sigma_{rp} = \{[V_{pr}^2]/N\}^{1/2}$; $\sigma_{\alpha p} = \{V_{p\alpha}^2/N\}^{1/2}$ and $\sigma_{p\delta} = \{V_{p\delta}^2/N\}^{1/2}$ represent the standard deviations of the peculiar velocity in the direction r, α and δ respectively. For σ_{pr}, $\sigma_{p\alpha}$ and $\sigma_{p\delta}$ minimum values will be obtained when the correct values for $V_\odot$ and $a(y)$ are inserted. This leads to the conditions

$$\partial\sigma_{pr}/\partial_\odot = 0 \; ; \; \partial\sigma_{pr}/a = 0 \; ; \; \partial\sigma_{p\alpha}/\partial V_\odot = 0 \; ; \; \partial\sigma_{p\alpha}/\partial a = 0 \; ;$$

$$\partial\sigma_{p\delta}/\partial V_\odot = 0 \quad \text{and} \quad \partial\sigma_{p\delta}/\partial a = 0$$

For the radial velocities these conditions lead to the equation

$$\left.\begin{aligned} [V_R\, y^2 R] + V_\odot [y^2 R\lambda_1] - [y^3 R^2] - a[y^4 R^2] &= 0 \\ [V_R\, \lambda_1] + V_\odot [\lambda_1^2] - [y\, R\, \lambda_1] - a[y^2 R\lambda_1] &= 0 \end{aligned}\right\} \cdots (3)$$

These equations must simultaneously be fulfilled and therefore it must be possible to solve these equations for both $V_\odot$ and $a(y)$. When these equations are also applied to the larger distances there will be an unoertainty in the determination of R because the higher derivates of the differential rotation have been neglected. Also the approximation [1] may not longer be adequate. When it is admitted that with the Cepheids the distances y are affected by an error in the zero point of the period - absolute magnitude curve $\Delta M = -5 \log f$ instead of y we must write $f \cdot y$ and the equations (3) now read

$$\left.\begin{aligned} [V_R\, y^2 R] + V_\odot [y^2 R\, \lambda_1] - f[y^3 R^2] - a f^2 [y^4 R^2] &= 0 \\ [V_R\, \lambda_1] + V_\odot [\lambda_1^2] - f[R\lambda_1] - a f^2 [y^2 R\, \lambda_1] &= 0 \end{aligned}\right\} \cdots (4)$$

and it is hardly possible to solve the three unknown qnantities $V_\odot$, $a(y)$ and f from these equations.

In a similar way the second of the equations (2) and the minimum conditions $\partial\sigma_{\alpha p}\partial V_\odot = 0$ lead to the equations

$$\left.\begin{aligned} f[y^3\mu_\alpha^2] + \frac{V_\odot}{4.74}[y^2\mu_\alpha\lambda_2] + af^2[y^4\mu_\alpha^2] = 0 \\ f[y\mu_\alpha\lambda_2] + \frac{V_\odot}{4.74}[\lambda_2^2] + af^2[y^2\mu_\alpha\lambda_2] = 0 \end{aligned}\right\} \quad \ldots (5)$$

while a similar set of equations is obtained for μ_δ. If for $V_\odot$ a certain value is inserted in these equations, they can be solved for the unknown quantities f and af^2, so that numerical values for both f and a are obtained. Obviously before solving for f and af^2 the equations for μ_α and μ_δ should be combined into one set. From the shape of the equations (5) it is evident that the resulting values of f are directly proportional to the numerical value which is inserted for $V_\odot$ while that of $a(y)$ is inversely proportional to $V_\odot$.

It is therefore sufficient if the equations (5) are solved for only one value of $V_\odot$.

§ 3. The observational material.

The stars which are used in the present computations are those for which the radial velocities and/cr the proper motions are given in Wilson's General catalogue of radial velocities [7] and the General Catalogue of proper motions. They are collected in tables I and II. Table I contains the Cepheids for which foto-electric magnitudes are available [1]. With the stars in table II the visual maagnitudes were taken from Wilson's Catalogue.

These visual magnitudes were reduced to the fotographic scale by applying mean corrections based on the values given by Eggen, Gascoigne and Burr in their Table IV [4].

When the visual magnitudes of all Cepheids in Wilson's catalogue are treated in the same way it appears that there is no systematic difference between the magnitudes obtained in this way and the foto-electric magnitudes or at least that the distribution of the individual differences is too large to determine a possible systematic trend.

Table I.

Stars used in the present investigation (first class determination of distance)

Design	$y/1000$	V_R km/sec	μ_α ″.001	μ_δ ″.001	λ_1 .001	λ_2 .001	λ_3 .001	R .0001
α UMi	0.05	−17.4	+47	−11	−793	+390	+463	−144
δ Cep	0.15	−16.8	+20	0	−802	−059	+590	− 80
β Dor	0.17	−10.5	− 2	− 2	−127	−543	−823	0
η Aql	0.17	−14.8	+11	− 9	−382	+457	+801	+147
ζ Gem	0.21	+ 6.7	− 4	− 1	+194	+748	−630	+ 75
X Sgr	0.22	−13.5	− 4	−15	+ 82	+826	+552	+ 5
W Sgr	0.24	−28.6	+ 7	− 7	+ 4	+847	+527	+ 5
RT Aur	0.26	+21.6	+ 4	−15	+ 72	+848	−509	+ 16
DT Cyg	0.27	0.0			−619	+ 86	+776	+ 67
SU Cas	0.28	− 6.5	+ 6	−13	−667	+702	+232	−155
FF Aql	0.29	−22.−	− 5	−10	−194	+203	+961	+158
AH Vel	0.30	+ 8.8	− 2	+ 1	+449	−220	−862	+ 43
T Vul	0.33	− 1.4	+ 4	− 1	−574	+102	+806	+ 91
l Car	0.37	+ 4.0	− 8	+ 7	+718	−228	−654	− 64
Y Sgr	0.38	− 3.2	+10	−14	− 42	+731	+677	+ 70
S Sge	0.41	− 9.9	+ 4	− 6	−396	+233	+882	+149
SZ Tau	0.46	− 3.2	−14	− 8	−349	+690	−626	− 5
SU Cyg	0.48	−35.8	+ 2	− 1	−367	+ 27	+925	+123
T Cru	0.53	−13.9	0	+ 4	+878	+271	−387	−133
R Mus	0.56	0.0	− 5	−16	−880	+171	−438	−130
U Aql	0.60	− 6.5	+20	+ 1	−311	+566	+760	+133
R Cru	0.61	− 4.1	+17	−19	+878	+277	−384	−136
U Sgr	0.66	− 2.0	−10	− 4	− 87	+733	+677	+ 69
V350 Sgr	0.77	+ 9.5			−149	+751	+637	+ 69
X Cyg	0.83	+ 9.8	− 7	− 2	−551	−009	+836	+ 70
T Mon	0.85	+32.0	+19	+ 3	+ 72	+571	−807	+110
Y Oph	0.86	− 5.3	+ 4	−12	+ 51	+559	+829	+ 98
YZ Sgr	0.98	+18.5			−163	+702	+690	+90
SS Sct	1.01	+14.0			−134	+586	+792	+123
ST Tau	1.04	+ 1.0			− 96	+667	−758	+69
T Vel	1.05	− 8.4	+43	−16	+538	−189	−819	+22
TT Aql	1.11	0.0			−238	+456	+855	+152

Table I. (Continued)

Design	$y/1000$	V_R km/sec	μ_α ″.001	μ_δ ″.001	λ_1 .001	λ_2 .001	λ_3 .001	R .0001
V496 Aql	1.19	+ 5.2			−224	+584	+776	+129
RX Cam	1.20	−35.0			−457	+770	−444	−147
RZ Vel	1.26	+ 8.9	−12	− 2	+526	−150	−818	+ 45
RX Aur	1.30	−21.0			−261	+897	−344	− 80
FM Aql	1.45	−12.0			−238	+319	+916	+160
BG Lac	1.54	−19.5			−774	+ 27	+668	− 16
Y Lac	1.58	−18.0			−768	− 41	+637	− 50
RR Lac	1.65	−35.0			−820	− 1	+567	− 85
XX Sgr	1.71	+ 2.5			− 57	+705	+702	+ 80
FN Aql	1.75	+ 8.0			−253	+426	+867	+158
Z Lac	1.80	−25.0			−820	− 11	+566	− 85
RY Sco	1.98	−17.5			+ 66	+880	+466	− 22
X Vul	2.02	−13.0			−408	+ 66	+903	+130
WZ Sgr	2.03	−11.0			− 26	+730	+679	+ 66
RS Pup	2.07	+19.0			+463	− 30	−885	+ 94
SV Per	2.25	− 9.5			−306	+905	−299	− 94
SZ Aql	2.48	+ 9.5			−209	+456	+863	+150
RW Cas	3.19	−62.5			+818	+526	+227	−157

Table II

Additional stars with less well determined distance parameter

Design	$y/1000$	V_R km/sec.	μ_α ″.001	μ_δ ″.001	λ_1 .001	λ_2 .001	λ_3 .001	R .0001
R Tra	0.47	—19.9	— 5	—22	+592	+780	—160	—157
S Tra	0.52	+ 2.0	— 7	— 7	+470	+878	— 94	—152
TU Cas	0.55	—21.7			—877	+349	+323	—133
RV Sco	0.57	—22.—	— 7	—27	+262	+860	+484	— 54
S Nor	0.62	— 3.—	0	— 5	+403	+915	+016	—141
S Cru	0.63		— 7	—10	+862	+397	—306	—141
V Cen	0.65		—24	—16	+715	+686	—118	—158
AP Sgr	0.67	—18.0	— 9	—11	—011	+777	[illegible]626	+ 38
W Gem	0.69	— 0.2	+12	0	+103	+681	—726	+ 90
VX Pup	0.74	+12.0			+311	+130	—943	+152
U TrA	0.74		+25	—20	+470	+878	— 64	—150
AP Pup	0.76	+42.0			+409	—142	—899	+ 85
BB Sgr	0.76	+ 7.5	+14	—32	—163	+736	+654	+ 72
ER Car	0.78		+ 9	—21	+853	+059	—511	—102
AW Per	0.79	+13.5			—306	+868	—374	— 75
RY CMa	0.81	+37.0			+253	+301	—916	+160
U Vul	1.93	—11.2	— 3	—12	—326	+164	+929	+150
AT Pup	1.02	+24.—			+463	—074	—883	+ 90
V Car	1.04		+ 3	—18	+514	—383	—764	+ 22
U Car	1.18		+18	+18	+826	—023	—560	— 99
RW Tau	1.21	+17.0			—390	+525	—756	+ 44
XX Cen	1.21		— 9	—26	+812	+544	—210	—157
RS Ori	1.34	+42.0			+ 57	+681	—727	+ 85
X Lac	1.37	—20.0			—831	+ 21	+553	— 85
VZ Cyg	1.51	—16.5			—736	+ 15	+674	— 11
V Lac	1.51	—20.0			—831	+ 21	+553	— 85
SY Aur	1.75	— 2.0			—217	+921	—299	— 85
WX Pup	1.89	+49.0			+354	+ 67	—934	+136
SV Mon	2.21	+26.5			+ 57	+573	—816	+115
SV Vul	2.43	— 2.5			+397	+ 66	+911	+130
X Pup	2.61	+61.5	+29	+23	+311	—142	—937	+150
CD Cyg	2.72	—11.0			—423	—044	+907	+102
RW Cam	2.79	—25.5			—496	+868	+032	—152
XX Cen	1.21		— 9	—26	+812	+544	—210	—157
AQ Pup	3.31	+41.—			+409	+039	—910	+123

Table III

Values of $V_{\odot}$ and $a(y)$ as derived from radial velocities

y	$f=1$		$f=2$		$f=3$		$f=4$		n
	$V_{\odot}$	$a.10^5$	$V_{\odot}$	$a.10^5$	$V_{\odot}$	$a.10^5$	$V_{\odot}$	$a.10^5$	
	A = + 0.016 Km/psc first class determinations only								
500	13.0	−493	13.0	−186	13 0	−111	13.0	−79	19
1000	17.0	+110	17.9	− 5	18.9	− 17	19.8	−17	30
1500	12 6	− 13	14.1	− 23	15.6	− 19	17.0	−16	39
2000	14.3	− 3	15.7	− 17	18.2	− 15	20.1	−13	45
2500	14.1	− 19	16.3	− 17	18.6	− 13	20.8	−10	50
	A = + 0.020 Km/psc. first class determination only								
500	12.8	−447	12.8	−168	12 8	−108	12.8	−77	19
1000	17.2	+ 62	18.3	− 17	19.7	− 22	20.8	−20	30
1500	12.9	− 27	14.8	− 27	16.7	− 21	18.5	−15	39
2000	14.8	− 15	17.2	− 20	19.6	− 16	22.1	−13	45
2500	14.6	− 25	17.4	− 19	20.2	− 14	23.2	−11	50
	A = + 0.016 Km/psc. all available stars								
1000	16.4	+ 42	16.7	− 25	17.1	− 26	17.6	−23	41
1500	15.2	+ 92	16.2	− 21	17.5	− 20	18.1	−17	54
2000	15.9	+ 6	17.3	− 16	18.7	− 15	20.0	−13	64
2500	16.0	− 13	17.8	− 15	19.6	− 12	21.3	− 9	74
	A = + 0.020 Km/psc. all available stars								
1000	16.4	− 3	16.7	− 34	17.5	− 30	18.0	−25	41
1500	15.5	− 11	16.7	− 26	17.9	− 22	19.1	−18	54
2000	16.2	− 9	18.0	− 20	19.8	− 17	21.6	−14	64
2500	16.5	− 20	18.7	− 16	20.9	− 12	23.1	−10	74

Table IV.

Values of f and a corresponding to $V_{\odot} = 20$ km/sec. For both the radial and tangential velocities the first set of values is based on first class determinations only, the second on all stars available. With the radial velocities the value A=+0.020 Km/psc. was adopted.

	Radial velocities				Tangential velocities					
y	$10^5 . a$	f	$10^5 . a$	f	$10^5 . a$	f	n	$10^5 . a$	f	n
500	—	—	—	—	—64	3.73	15			
690					—49	3.10	18			
1000	—22	3.31	—15	7.76	—41	3.50	24	—38	3.51	36
1260					—32	3.15	35	—42	2.42	40
1500	—15	4.72	—16	4.75						
2000	—16	3.17	- 16	3.08						
2500	—14	2.93	. 14	2.61						

Remarks to table IV: For other values of $V_{\odot}$ the values a and f as derived from the tangential velocities are $V/V_{\odot}$ and $V/V_{\odot}$ times the tabulated values. For the radial velocities see table III.

However, with a view to the results obtained by Eggen (1.c.) the values of y which appear in table II are considerably less reliable than those in table I. For computing the values λ_1, λ_2 and λ_3 the apex $\alpha = 271°$ and $\delta = +28°$ was adopted. The values R (l, b) were computed with A = + 0.016 Km/psc. and $l_0 = 328°$.

For A the value A = + 0.020 Km/psc. has been recommended by Oort and Morgan [8]. The computations have been carried out for both values of A but it is evident that the necessary corrections can directly be applied to the equations [4]. To the proper motions which appear in the tables both the corrections as suggested by Wilson and Raymond [9] and those for differential galactic rotation have been applied. It is to be observed that all Cepheid which belong to population II or are suspected as such were excluded from the tables.

§ 4. Numerical results derived from the radial velocities.

When the values of the coefficients as derived from the tables I and II are inserted in the equations [4], these can only be solved when either a certain value for f is adopted or $V_{\odot}$ is supposed to be known. If for f the values 1, 2, 3 and 4 respectively are adopted the results are as appear in table III. The upper part of this table gives the results which are obtained if only the stars in table I are used, the lower part those when also the stars of table II are included. With the latter no separate solution is given for stars with $y < 500$ parsecs because table II only contains one such star and the solution may therefore be taken to be identical to that given in the upper part. For the stars with $y < 500$ parsecs the determinant of the coefficients of f and af^2 happens to be very nearly equal to zero. So in this case it appears as if the numerical value of $V_{\odot}$ does not depend on the value f which is assumed. Actually this hardly is probable and with all other distances the numerical value of $V_{\odot}$ of depends on f and increases with increasing values of f. The variation of $a(y)$ with y is rather erratic. Of necessity the numerical value of $a(y)$ must be negative, but for $f = 1$ several values of $a(y)$ appear to be positive At the same time with $f = 1$ the scatter of the individual vslues $a(y)$ is very large. If f is increased the $a(y)$ values seem to attain a maximum negative value and afterwards there is a slow increase, the negative values becaming smaller On the whole the differences between the four sets of solutions in table IV are small, but still the results derived with A = +0 20 Km/psc. seem to be slightly more reasonable than those based on the value A = + 0 016. Consequently furher on only the values based on the A = + 0.020 will be used.

Conversely it is possible to adopt a certain value of $V_{\odot}$ and to compute the corresponding value of $a(y)$ and f. If the value $V_{\odot} = 20$ Km/sec., which seems reasonable, the results are as given in the left hand side of table IV. For obvious reasons no results could be obtained for the stars with $y < 500$ parsecs. If a smaller value of $V_{\odot}$ is adopted the resulting values of f will decrease while for $a(y)$ larger negative values will be found.

As a value of the solar velocity deviating very much from $V_\odot = 20$ Km/sec. is improbable, the resulting variations can be expected to be small. There seems to be a pronounced tendency of the numerical values of f to decrease when stars of ever increasing distance are included.

This is a rather embarassing point, which is fully discussed later on.

§ 5. Numerical results derived from the tangential velocities.

With the tangential velocities it is not necessary to carry out several solutions based on different values of f. As pointed out in section 2 the values f and $V_\odot$ are directly proportional to each other, while $a(y)$ and $V_\odot$ are inversely proportional. So it is sufficient if one solution is known. Consequently the value $V_\odot = 20$ Km/sec. was directly inserted in the equations (5). The results are indicated in table IV. As no proper motions are given for Cepheids with $y > 1260$, for the tangential velocities the limit $y = 1260$ is used instead of $y = 1500$. The values corresponding to $y = 690$ parsecs are derived from the tau and upsilon components of the proper motions of the 18 stars considered by Blaauw and Morgan [3]. The largest value of y among these stars is 690.

It is not necessary to repeat the data here, all relevant details are contained in their table. It is evident that with the tau component the values λ_2 in the equation 5 are zero, while with the upsilon component $\lambda_2 = \sin \lambda$ where λ is the angular distance to the solar apex.

§ 6. Discussion of the numerical results.

The values f and $a(y)$ of table IV are graphically represented in the figure 1 and 2. Black circles correspond to values derived from the radial velocities, the larger being based on first class determination only, the smaller on all stars avaiable. In the same way large and small open circles denote values derived from the tangential velocities. The values derived

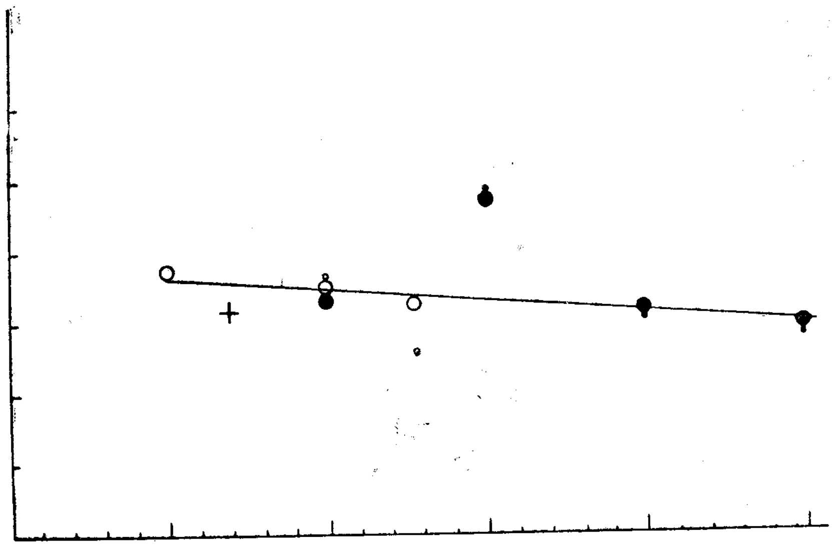

Fig. 1. Variation of f with y.

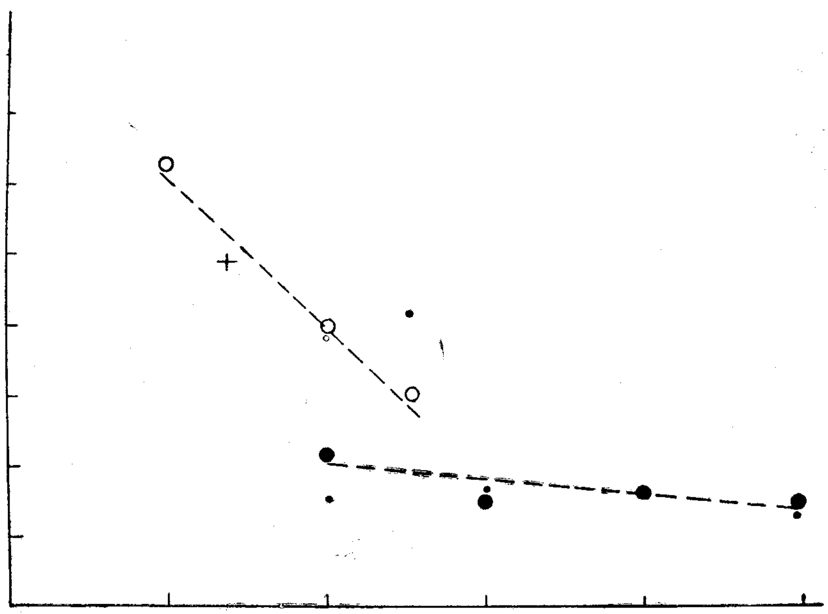

Fig. 2. Variation of a with y.

from the material of Blaauw and Morgan are indicated by a cross.

From figure 1 it is evident that there is a correlation between the values of f and y, the values of f decreasing with increasing value of y.

This correlation is roughly represented by the straight line in fig. 1. The agreement between the values of f derived from the radial velocities is satisfactory. The run of the value $a(y)$ in figure 2 can not be represented by one single straight line. At first there is rapid decrease in the absolute value of $a(y)$. derived from the tangential velocities. This first part of the curve is entirely determined by values derived from tangential velocities. With the larger values of y the gradient is much smaller, but this second part of the curve is determined by $a(y)$ values derived from the radial velocities. It is doubtful whether with the larger values of y, the first part of the curve which is based on the tangential velocities will level off sufficiently to fit smoothly on to the second part. The first question which must be considered is is whether these variations of f and $a(y)$ are real or are due to some systematic error. Variations of f and $a(y)$ to the amount as suggested by the figures 1 and 2, although not a priori impossible, are highly improbable.

Two effects have to be considered :

1. The effect of selection :
2. The effect of the distribution of the individual absolute magnitude around their mean.

§ 7. The effect of selection.

Only Cepheids of large apparent brightness have been used. It is unavoidable that therefore the sample is biased in favour of Cepheids of high absolute luminosity. That is to say that with the Cepheids of each period the selection has been in favour of those Cepheids of which the absolute magnitude is smaller than the mean absolute magnitudes corresponding to their periods. It is difficult to evaluate how large this effect could be. Judging from the various diagrams which Shapley has published for the Cepheids in the Magellanic clouds, the standard deviation in the distribution of the absolute magnitude must roughly be of the order of ± 0.4 magnitudes, but part of the scatter may be due to patches of obscuration in the clouds. In any case the variations of the absolute magnitudes are by no means negligible. Therefore it may be assumed, that the sample of galactic Cepheids which is considered in the mean is too bright. If therefore their apparent magnitudes are taken as a distance parameter while the absolute magnitude is considered as to be uniquelly determined by the period, in the mean the distance will be underrated.

When next the stars are arranged in order of increasing value of the distance paramnter y this arangement in the mean still roughly corresponds to a grouping of the stars in order of increasing distance. At the same time with decreasing values of y we include Cepheids with increasingly underevaluated distances. The result is that the f values which must be applied to convert the y values into true distances are not a constant but increase with decreasing value of y. It is evident that if with the smaller values of y the distances are generally underestimated, the absorption per unit distance is overestimated, at least if only the tangential velocities are used. As appears from table III, with the radial velocities the situation is more complicated and there may be an increase of the numerical values of $a(y)$. It is to be observed that the method exposed in section 2 must be very sensitive to the effects of selection, because the equations 4 and 5 contain terms up to the fourth power in y. But it seems doubtful that at the present time there is any method in which the effect of selection can entirely be avoided. In any case it must be concluded that in figures 1 and 2 the systematic variations of the values f and $a(y)$ are due to an effect of selection. The best values for f and $a(y)$ which can be obtained from our result are the ultimate values corresponding to $y = 2500$. With a sufficient degree of approximation for these we can write :

$$f = 3.0 \quad \text{and} \quad a(y) = -0.00016 \text{ magn./Kpc.}$$

but it is evident that this value of f represents a highest possible limit and that of $a(y)$ a lowest possible limit. These values of f and $a'y)$ correspond to $\Delta M = -2^m.40$ and $a(r) = +0.42$ magn./Kiloparsec.

The influence of the distribution of the individual absolute magnitudes around their mean values as determined by the period needs only briefly to be considered. To a deviation of $\pm \Delta M$ in the absolute magnitude there corresponds a range $y/y_0 = 10^{0.2\,\Delta M}$ of the distance parameter. It is a well known fact that if the mean value y is taken this mean value $\bar{y}$ is larger than y_0. There are standard procedures by which this can be accounted for, but these can only be applied if sufficient numbers of stars are avaiable, the samples are not biased and the standard deviation of ΔM is well known. Neither of these conditions are fulfilled and so even if the material were statistically more complete no reliable values for ΔM and $a(r)$ could be obtained. It is obvious that a probable error in the observed values of the apparent magnitude would tend to have the same effect as the distribution of the absolute magnitudes. The distance parameters would be too large in the mean, resulting in smaller values of f and larger values of $a(r)$. As a matter of fact when the procedure exposed in this paper was applied while using one of the older fotographic series of magnitudes, values for $\Delta M(f)$ and $a(r)$ were obtained which are far closer to the commonly accepted values. As the values of f and $a(r)$ obtained in such a way may be largely spurious it is useless to quote numerical results.

§ 8. Summary.

1. A method has been developed by which it should be possible simultaneously to determine the velocity of the sun and the coefficient of intersellar absorption from the radial velocities and from the proper motions.

2. A factor f is introduced to account for a possible error in the zero point of the absolute magnitude curve. It is however not possible to solve the three unknown quantities $V_{\odot}$, f and $a(y)$ from the equations.

3. In the equations terms occur containing the distance parameter y up to the fourth power. This makes the method rather sensitive to errors in distance in particular to those resulting from the effect of selection.

4. The method was applied to the bright galactic Cepheids but only an upper limit of $\Delta M = - 2.^m40$ and a lower limit of $a(r) = + 0.42$ magn/Kiloparsec could be determined.

5. The values of f and $a(y)$ obtained for different limits of the distance parameter y clearly show the influence of selection.

6. The stars which are available have been selected on account of their apparent brightness and their mean absolute magnitude must be brighter than the mean of the absolute magnitudes derived from their periods. This effect will be strongest with stars having a small distance parameter y.

7. The effect is that the numerical value of both f and $a(y)$ apparently depends on y.

8. It seems to the author that the main uncertainty in the determination of the zero point correction of the galactic Cepheids is the uncertainty of the effect of selection.

9. From the results in the present paper it is clear that selectional effects are present, but at present it is not possible to apply corrections to account for it.

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