A Resonance Method of $\varepsilon/\varepsilon_0$ and μ/μ_0 Measurement of Ferrites at Centimeter - Wavelength Region

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Özet: Bu çalışmada ferrite'lerin santimetrik dalgalar sahasında dielektrik sabiti ve permeabilitesini tayin etmek için bir rezonans metodu geliştirilmiştir. E_{010} - modu ile titreşen kaviteleri kullanan bu metod evvelâ matematik yoldan elde edilmiş olup sonra da experimentel olarak tatbik edilmiştir. Metodun doğruluk derecesinin, kavitenin rezonans frekansının tayini doğruluk derecesi ile çok sıkı ilgili olduğu gösterilmiştir. Her nekadar rezonans frekansının istenilen doğruluk derecesinde tayini, deneyin yapılması bakımından güç ise de yine de bu metodun iyi bir yaklaşık metod olduğu açıklanmıştır.

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Summary: The present paper deals with the theory and experimental development of a resonator system suitable for measurements of the dielectric constant and permeability of ferrites material in the wave - length region of 10cm (3000MC/sec). A method for measuring $\varepsilon_1/\varepsilon_0$ and μ_1/μ_0 using a cylindrical cavity - resonator operating in the E_{010} - mode at 10 cm wave - length region is calculated and some experiments are carried out to apply this method. It can be shown that the accuracy of the method is limited mainly by the accuracy of determination of the resonance frequency of the cavity. Although it is rather difficult to determine experimentally the resonance frequency to the desired accuracy, the method seems to be a good way to measure approximately $\varepsilon_1/\varepsilon_0$ and μ_1/μ_0 of ferrite materials with very small loss.

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I. INTRODUCTION

Ferrites are refractory materials composed of the oxides of iron and other metals such as Mn, Co, Ni, Cu, Zn, or Mg. When prepared by pressing the powder oxides into the required shape and fired, they become hard and strong. In this form many interesting applications have already been found, especially in the high frequency field. Besides this, ferrites are very interesting research materials for science and engineering. When investigating these materials it is very often desired to measure their dielectric constant and permeability. Although there are some approximate methods for this purpose, none of them uses a resonance method. It was desirable therefore to develop a suitable resonance method for ferrites. The procedure which will be described here, is based on the use of two resonant cavities, operating in the E_{010} - mode.

II. THE CALCULATION OF THE METHOD

The E_{010} - mode is characterised, for perfectly conducting boundaries, by a purely longitudinal electric field and a purely circomferential magnetic field. The theory of a cylindrical resonator, operating in this mode, is fully described by Willis Jackson and his colloborators[1]. They solved the Maxwell equations and obtained the field equations for completely and partly dielectric filled cavities.

To determine the permittivity ε they drived a relation from the field equation of the cavity. Their equation is given below :

$$\frac{\varepsilon_1}{\varepsilon_0}=\frac{\beta_1}{\beta_0}\times\frac{J_0(\beta_1 b)\,J_1(\beta_0 b)}{J_1(\beta_1 b)\,J_0(\beta_0 b)}\left\{\frac{\dfrac{Y_0(\beta_0 a)}{J_0(\beta_0 a)}-\dfrac{Y_1(\beta_0 b)}{J_1(\beta_1 b)}}{\dfrac{Y_0(\beta_0 a)}{J_0(\beta_0 a)}-\dfrac{Y_0(\beta_0 b)}{J_0(\beta_0 b)}}\right\}$$

Whre ε_1, ε_0 are the permittivities of the specimen and of the free space respectively in terms of rationalised M. K. S. units $\beta_0=\dfrac{2\pi}{\lambda_0}$ is the phase canstant for free space, λ_0 being the resonant free-space wav elength corresponding to the frequency ω_0.

β_1 is the phase constant in the specimen region and is given by the following equation :

$$\beta_1 = \beta_0 \sqrt{\frac{\varepsilon_1}{\varepsilon_0} \times \frac{\mu_1}{\mu_0}} .$$

where μ_0 and μ_1 are the permeability of free space and of the specimen respectively.

a and b are the radii of the resonant cavity and of the specimen.

J_0, J_1 are Bessel Functions of the first kind and Y_0, Y_1 are functions of the second kind.

Our purpose was to apply this resonance method to determine $\varepsilon_1/\varepsilon_0$ and μ_1/μ_0 of ferrite materials. Since in this case we have at least two unknowns to determine it is necessary that two different measurements be made in order to establish two independent relationship for the one specimen. These experiments are made on two different cavities having different radii but operating in the same mode (E_{010}).

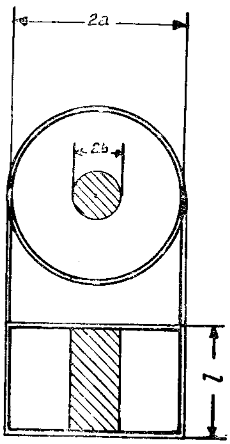


Fig. 1. Cavity of the radius a with the specimen

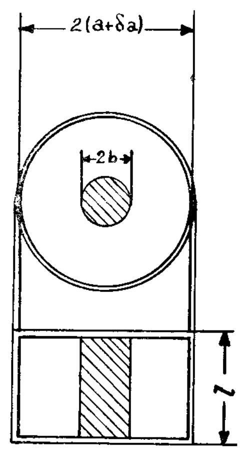


Fig. 2. Cavity of the radius $(a + da)$ with the speeimen

For the firsit cavity of radius (a) which contains the specimen of radius (b) as shown in Fig. 1, we can apply Jackson's equation which is given below in a rearranged form.

$$\frac{\beta_0}{\beta_1} \times \frac{\varepsilon_1}{\varepsilon_0} = \frac{J_0(\beta_1 b)\,[Y_0(\beta_0 a)\, J_1(\beta_0 b) - Y_1(\beta_0 b)\, J_0(\beta_0 a)]}{J_1(\beta_1 b)\,[Y_0(\beta_0 a)\, J_0(\beta_0 b) - Y_0(\beta_0 b)\, J_0(\beta_0 a)]} \quad (1)$$

$$= F(\beta_0, \beta_1, a, b)$$

Knowing the value of β_1, it is then possible to calculate by the above equation the relative permittivity. The relative permeability $\frac{\mu_1}{\mu_0}$ then can be determined from the equation

$$\beta_1 = \beta_0 \sqrt{\frac{\varepsilon_1}{\varepsilon_0} \times \frac{\mu_1}{\mu_0}}$$

To find an equation which gives β_1 we make use of another cavity of radius $(a + da)$ as shown in Fig. 2. The resonant frequency of this cavity will be lower by the amount $d\omega$ so that we have in this case a cavity of radius $(a + da)$ and resonant frequency $(\omega - d\omega)$. If the change in the resonant frequency is kept very small we may assume that ε and μ of the specimen do not change appreciably so that the left hand side of the equation (I) remains unaltered. Therefore it is permissible to write the differential of equation (I) as given below in equation 2.

$$da \frac{\partial F}{\partial a} - d\omega \frac{\partial F}{\partial \omega} = 0 \tag{2}$$

$\frac{\partial F}{\partial a}$ and $\frac{\partial F}{\partial \omega}$ can be calculated in the following way:

$$\frac{1}{F} \times \frac{\partial F}{\partial a} = \frac{\beta_0[Y_0(\beta_0 a)J_1(\beta_0 b) - J_0'(\beta_0 a)Y_1(\beta_0 b)]}{Y_0(\beta_0 a)J_1(\beta_0 b) - J_0(\beta_0 a)Y_1(\beta_0 b)} - \frac{\beta_0[Y_0'(\beta_0 a)J_0(\beta_0 b) - J_0'(\beta_0 a)Y_0(\beta_0 b)]}{Y_0(\beta_0 a)\, J_1(\beta_0 b) - J_0(\beta_0 a)\, Y_0(\beta_0 b)}$$

By using the identities

$$J_0' = - J_1 \; ; \; Y_0' = - Y_1$$

and by substituting

$$\beta_0 a = x \; ; \; \beta_0 b = y$$

the following equation can be obtained:

$$\frac{1}{F} \times \frac{\partial F}{\partial a} = \frac{\beta_0[J_0(y)Y_1(y) - J_1(y)Y_0(y)]\,[Y_0(x)J_1(x) - Y_1(x)J_0(x)]}{[Y_0(x)J_1(y) - Y_1(y)J_0(x)]\,[Y_0(x)J_0(y) - Y_0(y)J_0(x)]}$$

By making of use of the following identities

$$Y_1(y)\, J_0(y) - Y_0(y)\, J_1(y) = - \frac{2}{\pi y}$$

$$Y_0(x)\, J_1(x) - Y_1(x)\, J_0(x) = \frac{2}{\pi x}$$

the above given equation can be rearranged as shown below in equation (3)

$$\frac{1}{F} \times \frac{\partial F}{\partial a} = \frac{-4\beta_0}{\pi^2 xy\,[Y_0(x)J_1(y) - Y_1(y)J_0(x)]\,[Y_0(x)J_0(y) - Y_0(y)J_0(x)]} \tag{3}$$

To calculate $\frac{\partial F}{\partial \omega}$ use will be made again of the logarithmic differentiation of Jackson's equation:

$$\begin{aligned} \frac{1}{F} \times \frac{\partial F}{\partial \omega} &= \frac{J_0'(\beta_1 b)}{J_0(\beta_1 b)} \times \frac{\beta_1 b}{\omega} - \frac{J_1'(\beta_1 b)}{J_1(\beta_1 b)} \times \frac{\beta_1 b}{\omega} \\ &- \frac{[Y_0'(\beta_0 a)J_1(\beta_0 b) - Y_1(\beta_0 b)J_0'(\beta_0 a)]\frac{\beta_0 a}{\omega} + \frac{\beta_0 b}{\omega}[Y_0(\beta_0 a)J_1'(\beta_0 b) - Y_1'(\beta_0 b)\, J_0(\beta_0 a)]}{[Y_0(\beta_0 a)J_1(\beta_0 b) - Y_1(\beta_0 b)J_0(\beta_0 a)]} \\ &+ \frac{[Y_0'(\beta_0 a)\, J_0(\beta_0 b) - Y_0(\beta_0 b)\, J_0'(\beta_0 a)]\frac{\beta_0 a}{\omega} + [Y_0(\beta_0 a)\, J_0'(\beta_0 b) - Y_0'(\beta_0 b)\, J_0(\beta_0 a)]\frac{b\beta}{\omega}}{[Y_0(\beta_0 a)J_0(\beta_0 b) - Y_0(\beta_0 b)J_0(\beta_0 a)]} \end{aligned} \tag{4}$$

The existence of $\left(\frac{1}{\omega}\right)$ in the above equation can be explained if we think that (β_1) and its derivative with respect to ω are given as follow:

$$\beta_1 = \frac{\omega}{c}\sqrt{\frac{\varepsilon_1}{\varepsilon_0} \times \frac{\mu_1}{\mu_0}}$$

$$\frac{d\,\beta_1}{d\,\omega} = \frac{1}{c}\sqrt{\frac{\varepsilon_1}{\varepsilon_0} \times \frac{\mu_1}{\mu_0}} = \frac{\beta_1}{\omega}$$

If we substitute $\beta_0 b = y$, $\beta_0 a = x$ and $\beta_1 b = Z$ and introduce the identities

$$J_1(x)Y_0(x) - Y_1(x)J_0(x) = \frac{2}{\pi x}$$

$$Y_1(y)J_0(y) - Y_0(y)J_1(y) = -\frac{2}{\pi y}$$

we can obtain, after rearranging the equation 4, the final equation for the calculation of the value of $\left(\frac{\partial F}{\partial \omega}\right)$. This equation is given below:

$$
\begin{aligned}
&\frac{\omega}{F} \times \frac{\partial F}{\partial \omega} = 1 - Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right] \\
&- \Big\{ \frac{-4}{\pi^2 y} - 2y Y_0(x) J_0(x) [J_1(y) Y_1(y) + J_0(y) Y_0(y)] \\
&+ Y_0(x) J_0(x) [Y_0(y) J_1(y) + J_0(y) Y_1(y)] + Y_0^2(x) y [J_0^2(y) + J_1^2(y)] \\
&- Y_0^2(x) J_0(y) J_1(y) + y J_0^2(x) [Y_0^2(y) + Y_1^2(y)] - J_0^2(x) Y_0(y) Y_1(y) \Big\} \\
&\{ [Y_0(x) J_1(y) - Y_1(y) J_0(x)][Y_0(x) J_0(y) - Y_0(y) J_0(x)] \}^{-1}
\end{aligned} \tag{5}
$$

If we multiply the equation (3) by $(da \times \omega)$ and the equation (5) by $(d\omega)$ it is then possible by the aid of the equation (2) to set up the following relationship :

$$(da \times \omega) \times \text{Equation (3)} = (d\omega) \times \text{Equation (5)} \tag{6}$$

Since the values of da, ω, $d\omega$ can be measured it is possible to evaluate the term

$$Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right]$$

in the equation (5).

Suppose we make a table giving the numerical values of this term for different values of Z. Then knowing from experimental data the value of this expression we will be able to determine the value of Z and hence the value of (β_1).

After inserting this value of (β_1) into the equation (1), one can evaluate the relative permittivity $\varepsilon_1/\varepsilon_0$. Then from the equation

$$\beta_1 = \beta_0 \sqrt{\frac{\varepsilon_1}{\varepsilon_0} \times \frac{\mu_1}{\mu_0}}$$

it is possible to calculate the relative permeability μ_1/μ_0.

III. EXPERIMENTAL PART

To make this calculation clear we shall apply this method to an example of measurement :

The material to be measured was(Ferrox cube B_2), B_2 being the trade name of this ferrite. Its properties and composition were not known to us. Table I gives us the resonant wavelength of the two different cavities each oscillating in the E_{010} - mode, without specimen.

Table I.

Free space resonant wavelengths and radii of the cavities.

Cavity	λ_0 in meter	Radius a in meter
I	9.832×10^{-2}	3.572×10^{-2}
II	9.178×10^{-2}	3.509×10^{-2}

The resonant wavelengths were determined with the aid of the resonance curves (see Fig. 3 and 4). The radii were calculated from the following equation :

$$\lambda_0 = 2.6125\, a$$

which can be obtained from the field equation of a resonant cavity operating in the E_{010} - mode. The values obtained in this way agree with the actual physical values which indicates that the resonator oscillates definitely in the E_{010} - mode. We find from these values the difference in the radii to be

$$da = (a_1 - a_2) = 0.063 \times 10^{-2} \text{ meter.}$$

To obtain the resonance curves experimentally, a klystron CV 67 was used as a variable frequency oscillator, its frequency being varied by a fine tuner and measured by a frequency meter designed by L. Essen and A.C. Cordon[2].

The ferrite specimen to be measured was inserted in the center of each cavity successively and the resonance curves were plotted. Great care was used to keep all conditions unaltered such as the coupling factor which affects the frequency of the resonator. The resonance curves of these partly filled cavities are given in Fig. 5 and 6. In the table II we have the resonant wavelengths and corresponding frequencies for the cavities containing a ferrite rod of the radius $b = 0.001718$ m.

Table II.

Resonant wavelengths and corresponding frequencies of partly ferrite filleds cavities

Cavity	λ in meter	$\omega/2\pi$ MC.
I	9.741×10^{-2}	3079.576
II	9.586×10^{-2}	3129.466

$$\frac{d\omega}{2\pi} = 49.890$$

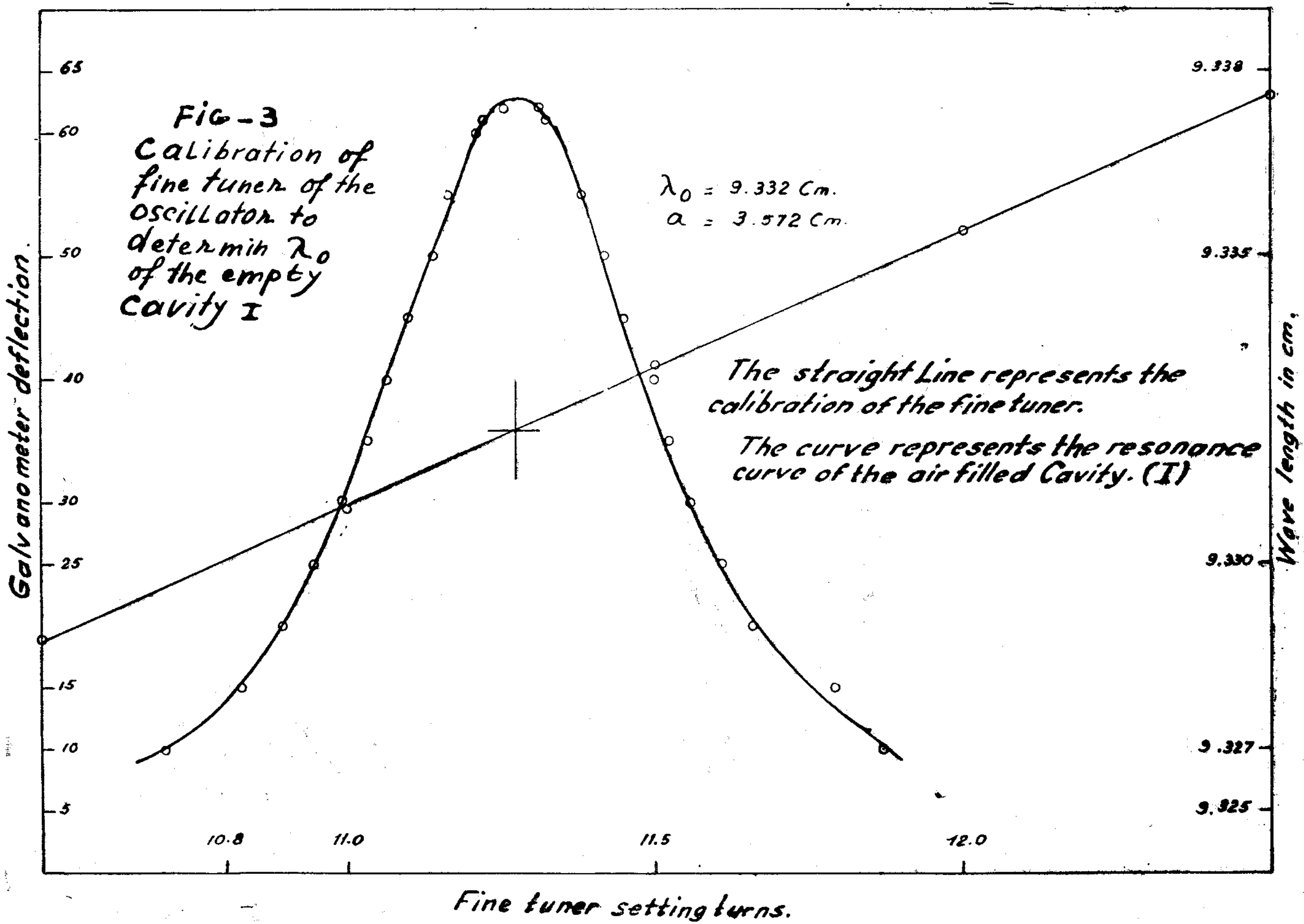


Fig - 3
Calibration of fine tuner of the oscillator to determin λ_0 of the empty Cavity I

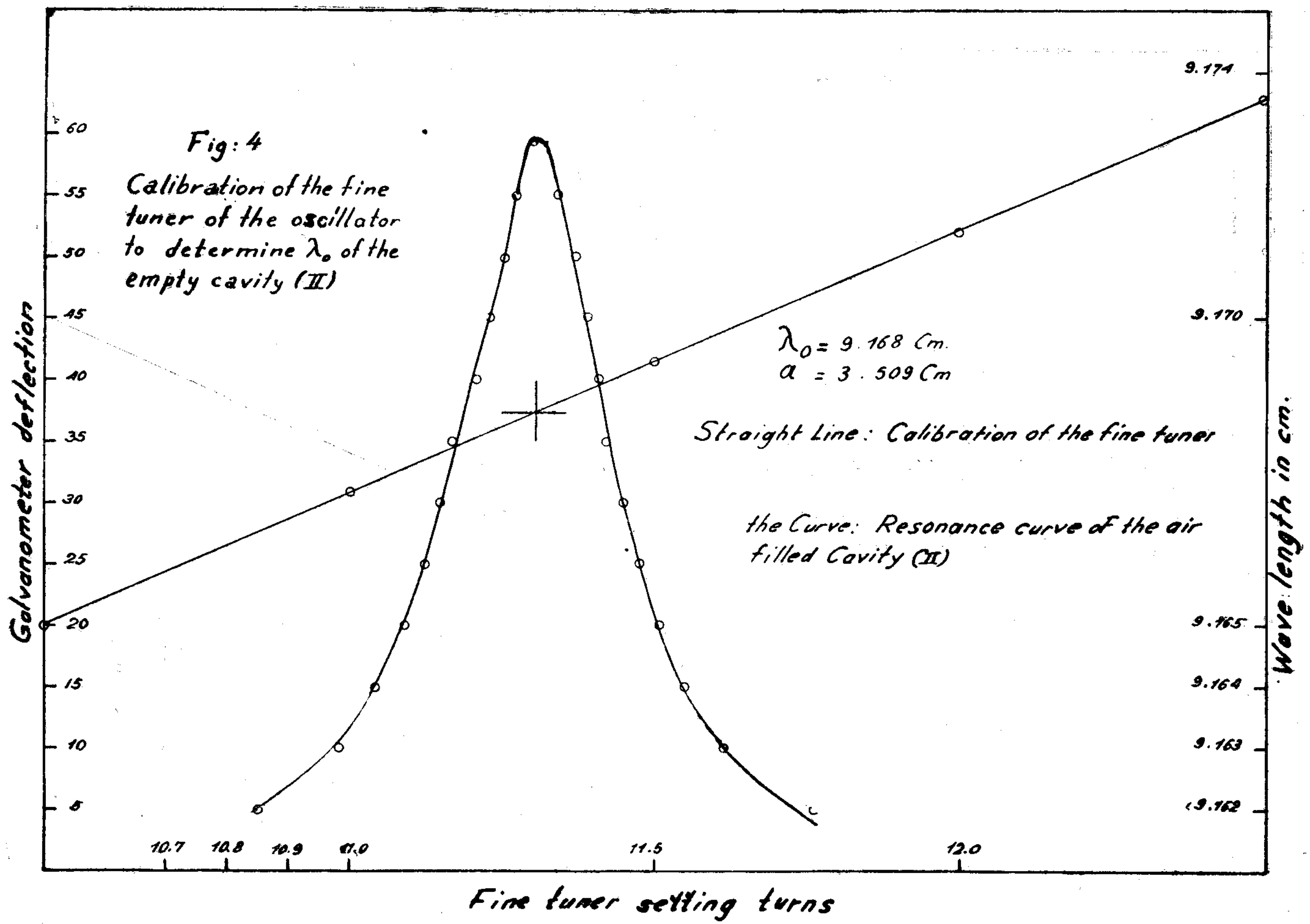


Fig: 4
Calibration of the fine tuner of the oscillator to determine λ_o of the empty cavity (II)
$\lambda_0 = 9.168$ Cm.
$a = 3.509$ Cm
Straight Line: Calibration of the fine tuner
the Curve: Resonance curve of the air filled Cavity (II)
Galvanometer deflection
60
55
50
45
40
35
30
25
20
15
10
5
Wave length in cm.
9.174
9.170
9.165
9.164
9.163
9.162
10.7
10.8
10.9
11.0
11.5
12.0
Fine tuner setting turns

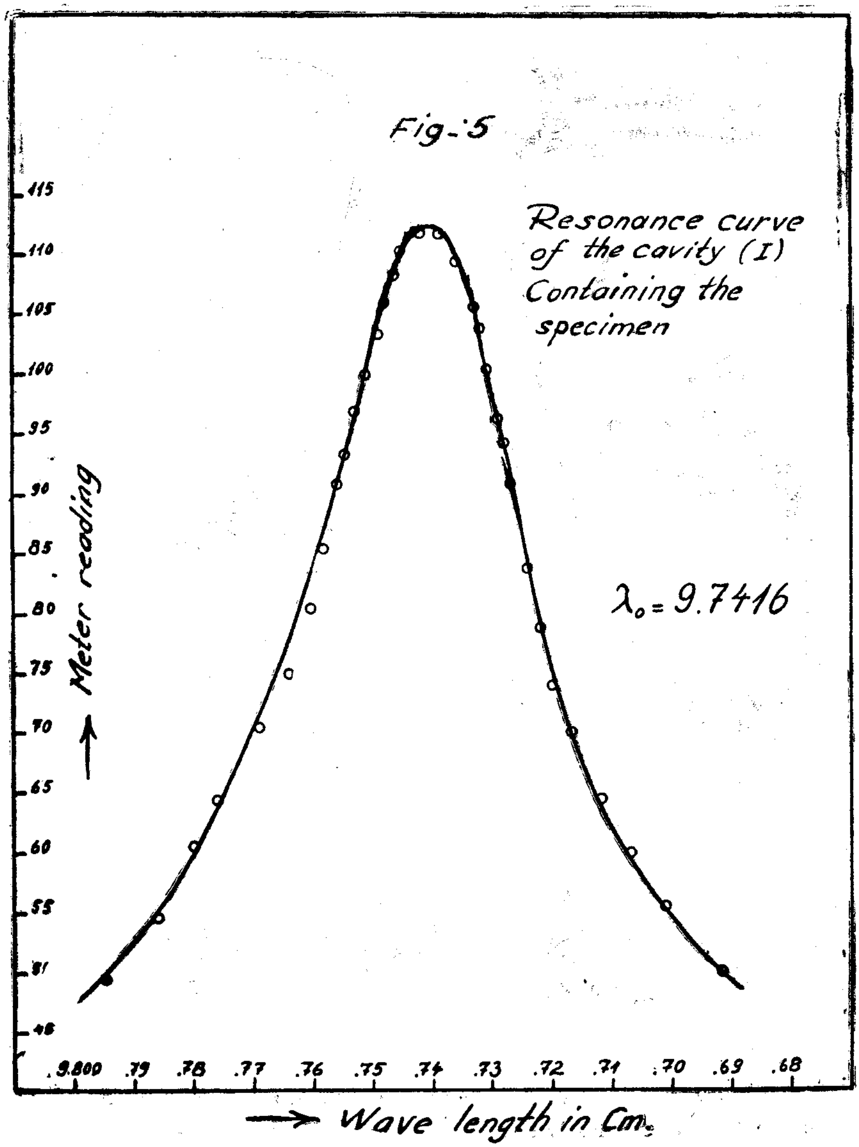
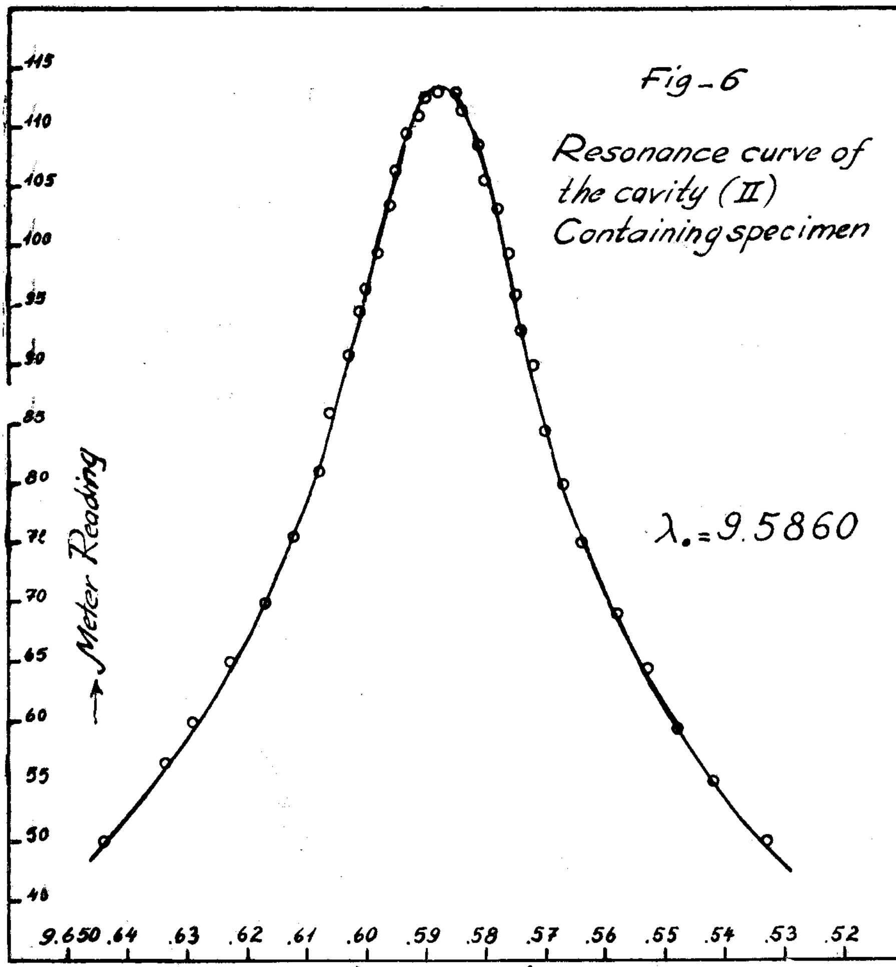


Fig.-5
Resonance curve of the cavity (I) Containing the specimen
$\lambda_o = 9.7416$
Meter reading
115
110
105
100
95
90
85
80
75
70
65
60
55
51
48
9.800
.79
.78
.77
.76
.75
.74
.73
.72
.71
.70
.69
.68
Wave length in Cm.



It is now possible to compute the equations (3) and (5) with reference to Table III which contains the experimental results necessary for the calculation.

The results obtained by carriying out the calculations are given below :

$$\frac{1}{F} \times \frac{\partial F}{\partial a} = -\,487.323$$

Table III.

The experimental values obtained from measurements.

b 10^{-2} m.	a 10^{-2} m.	da 10^{-2} m.	$\omega/2\pi$ MC	$d\omega/2\pi$ MC	$\beta_0 = \frac{2\pi}{\lambda_0}$	$x = \beta_0 a$	$y = \beta_0 b$
0.1718	3.509	0.063	3129.466	49.890	65.545	2.299	0.1126

and $$\frac{\omega}{F} \times \frac{\partial F}{\partial \omega} = -Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right] - 16.882$$

To determine (Z) we proceed as follow :

$$(\omega \times da) \times \frac{1}{F} \times \frac{\partial F}{\partial a} = -\,0.3021 \times \omega$$

$$d\omega \times \frac{\omega}{F} \times \frac{\partial F}{\partial \omega} = d\omega \left\{ -Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right] - 16.882 \right\}$$

By referring to the relationship (6) we may write

$$-\,0.3021 \times \frac{\omega}{d\omega} + 16.882 = -\,Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right]$$

And because $\frac{\omega}{d\omega} = 62.727$, it is then

$$Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right] = 2.067$$

To find out the phase constant β_1 from $Z = \beta_1 b$ a table giving the values of

$$Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right]$$

as a function of (Z) was prepared first. A small part of this table, which is of interest for this experiment, is given below :

Z	$Z\left[\frac{J_0(Z)}{J_1(Z)} + \frac{J_1(Z)}{J_0(Z)}\right]$
0.47	2.9576
0.48	2.0600
0.49	2.0633
0.50	2.0656
0.51	2.0686
0.52	2.0715
0.53	2.0745

The value of (Z) is found to be 0.506. Hence the phase constant β_1 is

$$\beta_1 = Z/b = 0.506/0.001718 = 294.52$$

By inserting this value of β_1 into the equation (1) we obtain for the relative permittivity, or dielectric constant ;

$$\varepsilon_1/\varepsilon_0 = 10.03$$

accordingly the relative permeability will be

$$\frac{\mu_1}{\mu_0} = 2.01$$

Because the properties of this commercial ferrite B_2 were unknown to us, it was not possible to compare these experimental results with those measured by other methods. However a calculation of the relative permittivity of this specimen, assuming it to be as pure dielectric, gave approximately (10) which agrees quite well with our result.

IV. Conclusion

As described above the method seems to be suitable to determine the dielectric constant and the permeability of the ferrites having a small loss factor. Because the method is very sensitive to the factor $\omega/d\omega$ the accuracy of this method depends upon how accurate the resonant frequency can be determined, which in turn seems to be rather difficult from the experimental point of view. Therefore this method should be used only on ferrites with very small loss factor.

References

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