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Outline of a Method for Determining the Absorption in the Surroundings of the Sun

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Özet: Aşağıda, rasat edilen Spektroskopik paralakslar, radyal hızlar ve öz hareketlerden ortalama ekstiriksiyon ve ortalama uzay absorpsiyon kat-katsayısını tayin için bir metod sunulmuştur. $\Phi(\dot{x}, \dot{y}, \dot{z}/S)$ dağılımının güneşin hemen civarında sabit olduğu farzedilmiştir. Bu makul bir faraziyedir. Bu sebepten, hız kanunu doğrultulara bağlı olmayıp herhangi bir eksen boyunca sabit olmalıdır. Gök yüzünün muhtelif bölgelerinde eksen olarak yarıçap vektörü ve α ve δ dairelerine teğet ve dik doğrultuları seçmek uygundur. a yı ihtiva eden hareket denklemleri çıkarılır. Bunlardan a nın nümerik değeri kolayca bulunabilir.

* * *

Abstract: In the following a method is proposed to determine the mean coefficient of interstellar absorption and the mean extinction from the observed spectroscopic parallaxes, the radial velocity and the proper motions. The distribution $\Phi(\dot{x}, \dot{y}, \dot{z}/S)$ is assumed to be constant in the immediate surroundings of the sun. This is a reasonable assumption. Therefore the velocity law must be a constant along any axis, independent of its orientation. In different areas of the sky it is convenient to adopt as axis the radius vector and the direction perpendicular to and tangent to the circles α and δ. The equations of motion containing a are derived. From these the numerical values of a can easily be derived.

* * *

1. Introduction.

In the following it is supposed that in the immediate vicinity of the sun the distribution function Φ of the velocities is a constant for each spectral subgroup on the main branches of the H.R. diagram. So

$$\Phi = \Phi(\dot{x}, \dot{y}, \dot{z} \mid S) = C \quad \cdots\cdots (1)$$

where the vertical bar before S indicates that the spectral type S is to be taken as a constant parameter. For the individual stars the velocity components $V_x = \dot{x}$; $V_y = \dot{y}$ and $V_z = \dot{z}$ can be computed from the observed components of the proper motion and from the observed radial velocity, after these have been freed from possible systematic errors and from the effects of differential galactic rotation. Obviously for converting the proper motions into linear velocities the parallax of the star must be given. It is proposed only to use spectroscopic parallaxes.

The distances y corresponding to the spectroscopic parallaxes are obtained by comparing the absolute and the apparent magnitudes of the star while the absorption is neglected. It is obvious that in this way the distance of a star will be overestimated.

Let y be the apparent distance obtained from the spectroscopic parallax and r the true distance of the star. We have

$$r = y\, e^{\int_0^y \alpha(y)\, dy} = y e^{\bar{a}.y} \quad \cdots\cdots (2)$$

As we will mainly be concerned with small distances instead of (2) we may write

$$r = y(1 + a y) \quad \cdots\cdots (3)$$

It is evident that a can only have negative numerical values. With the aid of the values (3) the space velocities can be computed. These space velocities contain the term a, the mean absorption coefficient.

The numerical value of a must be such that the fundamental relation $\Phi = C$ is fulfilled.

The numerical value of a is to be determined from this condition. The necessary observational data are to be taken from the following sources:

«General Catalogue of proper motions» B. Boss, Carnegie Institution, 1936;
«General Catalogue of radial velocities» R. E. Wilson, Publ. 601, Carnegie Inst., 1953;
«General Catalogue of stellar parallaxes» P. Schlesinger, Yale Obs., 1935.

2. The corrections to be applied to the radial velocities.

The generalised Oort formulae for the differential effects of galactic rotation are

$$\left.\begin{aligned} \Delta V_R &= Ar \cos^2 b \sin 2 (l - l_0) \cdots\cdots (a) \\ \Delta K \mu_l &= B + A \cos 2 (l - l_0) \cdots\cdots (b) \\ \Delta R \mu_b &= -\frac{1}{2} A \sin 2 b \sin 2 (l - l_0) \cdots (c) \end{aligned}\right\} \cdots\cdots (4)$$

where A = + 0,016 km/sec parsec; B = − 0 014 km/sec parsec

$$K = 4{,}734, \quad l_0 = 328°.$$

For the present we only need the equation 4(a), but (b) and (c) are included for future reference.

The correction $-\Delta V_R$ must be applied to the observed radial velocities in order to eliminate the effects of galactic rotation. To these observed velocities there must also be applied the correction resulting from the constant K term. This latter correction can be neglected for all spectral types except the types O — B for which it is — 5 Km/sec.

In the following we put $V_R = V_R(\text{observed}) + K$

From (3) and (4) we have

$$V_R(\text{theor}) = \{V_R - Ay \cos^2 b \sin 2 (l - l_0) - aAy^2 \cos^2 b \sin 2 (l - l_0)\}$$

or, writing this in a simpler way

$$V_R(\text{theor}) = V_{R,y} - a y^2 R(l, b) \qquad \cdots\cdots (5)$$

where $V_{R,y}$ is the radial velocity corrected for the differential effect of galactic rotation corresponding to the apparent distance y.

3. Determination of a from the radial velocities.

If only stars of a given spectral type in a restricted area of the sky are considered, the equation (5) enables us to determine a. The radial velocity of an individual star must be equal to the opposite of the component of the solar velocity in radial direction plus the component of the star's peculiar velocity in that direction. So

$$-V_{R\odot} + V_{RP} = V_{RY} - ay^2 R(l, b) \qquad \cdots\cdots (6)$$

Each star yields an equation of the form (6). To solve these equations we can proceed in two different ways.

1. The component $V_{R\odot}$ is computed from the known space velocity of the sun relative to the group of stars under consideration. Summing all equations we will have $[V_{Rp}] = 0$ and $[V_{R\odot}] = N \cdot V_{R\odot}$ therefore we immediately find

$$a = \frac{N V_{R\odot} + [V_{R,y}]}{[y^2 R(l, b)]} \qquad \cdots\cdots (7)$$

This is the safest way to solve the equations (6).

2. The alternative is that $V_{R\odot}$ is considered to be an unknown constant. The stars are arranged in groups in the distance intervals $y \pm \frac{1}{2} dy$. For the stars in each group the equations are summed, so that from each interval we obtain an equation of the form

$$-V_{R\odot} = \overline{V}_{R,y} - a y^2 R(b, l)$$

Next $V_{R\odot}$ and a are found from a least squares solution, the normal equations being

$$\left.\begin{array}{l} V_\odot[-y^2 R(b, l)] + a[y^4 R^2(b, l)] = [-\overline{V}_{R,y} \cdot y^2 R(b, l)] \\ V_\odot[1] + a[-y^2 R(b, l)] = [\overline{V}_{Ry}] \end{array}\right\} \cdots\cdots (8)$$

The number of stars in a given limited area of the sky for which both the radial velocities and the spectroscopic parallaxes are known must be small. In the alternative method it is supposed that in each distance interval $[V_{Rp}] = 0$. But it is hardly to be expected that with small numbers of stars the peculiar velocities will cancel. Therefore it will not be possible to obtain reliable values for both $V_{R\odot}$ and a from the solution of the equation (8).

4. The corrections to be applied to μ''_α and μ''_δ

It has been shown by Wilson and Raymond[1] that the proper motions in the General Catalogue are subject to systematic errors. They suggest that the following corrections should be applied

$$\left.\begin{aligned} \Delta_1 \mu''_\alpha &= \Delta_1 (15\ \mu^s_\alpha \cos\delta) = \\ &\qquad +\cdot''0024 \cos\delta - 0\cdot''0037 \sin\alpha \sin\delta \\ \Delta_1 \mu''_\delta &= -\cdot''0038 \cos\delta \end{aligned}\right\} \cdots\cdots (9)$$

Further the observed proper motions are to be corrected for the effects of differential galactic rotation. The relations 4 a, b give these corrections for the components of proper motion in galactic longitude and galactic latitude. From this the corrections to be applied to the components of proper motions in right ascension and declination can be computed[2]. These corrections appear to be

$$\left.\begin{aligned} \Delta_2 \mu''_\alpha &= -\left[\frac{A}{4.74}\cos 2(l-l_0) + \frac{B}{4.74}\right]\cos b \cos\varphi - \\ &\qquad -\frac{1}{2}\frac{A}{4.74}\sin 2(l-l_0)\sin 2b \sin\varphi \\ \Delta_2 \mu''_\delta &= -\left[\frac{A}{4.74}\cos 2(l-l_0) + \frac{B}{4.74}\right]\cos b \sin\varphi + \\ &\qquad +\frac{1}{2}\frac{A}{4.74}\sin 2(l-l_0)\sin 2b \cos\varphi \end{aligned}\right\} \cdots\cdots (10)$$

where

$$A/4.74 = +\cdot''0038\,; \qquad B/4.74 = -\cdot''0027$$

and φ = galactic parallactic angle.

The values l, b and φ can be taken from Ohlsson's table in *Lund Annals 3* (1932).

In the following μ_α and μ_δ indicate the corrected values for the two components of proper motion expressed in seconds of arc

$\mu_\alpha = \mu_\alpha\,(\text{obs}) + \Delta_1 \mu_\alpha + \Delta_2 \mu_\delta$ and $\mu_\delta = \mu_\delta\,(\text{obs}) + \Delta_1 \mu_\delta + \Delta_2 \mu_\delta$

5. Determination of *a* from the proper motion.

For any individual star we have

$$- V_{\alpha\odot} + V_{\alpha p} = R\mu_\alpha r \quad \text{and} \quad - V_{\delta\odot} + V_{\delta p} = R\,\mu_\delta r$$

Inserting the value (3) in these equations we find

$$\left.\begin{aligned} - V_{\alpha\odot} + V_{\alpha p} &= K\,\mu_\alpha y + a\,K\,\mu_\alpha y^2 \\ - V_{\delta\odot} + V_{\delta p} &= K\,\mu_\delta\, y + a\,K\,\mu_\delta\, y^2 \end{aligned}\right\} \quad \dots\dots (11)$$

For each individual star we obtain a set of equations of this shape. Supposing again that only the stars of a given spectral type in a limited area of the sky are considered, we have the possibilities of two solutions for the equations (11).

1. The components $V_{\alpha\odot}$ and $V_{\delta\odot}$ of the solar velocity are computed from the known space velocity of the sun with respect to the stars of the spectral type under consideration. These values are inserted in the relations (11) and all equations are summed. We may expect that $[V^{\alpha p}] = [V_{\delta p}] = 0$. Therefore *a* is found from the equations

$$a = \frac{-\left(\frac{N}{K} V_{\alpha\odot} + [\mu_\alpha y]\right)}{[\mu_\alpha y^2]}; \qquad a = \frac{-\left(\frac{N}{K} V_{\delta\odot} + [\mu_\delta\, y]\right)}{[\mu_\delta\, y^2]}$$

However the best value for *a* will be obtained if we take the sums of the two equations (11) and compute *a* from

$$a = \frac{-\frac{N}{K}(V_{\alpha\odot} + V_{\delta\odot}) - [\mu_\alpha y] - [\mu_\delta y]}{[\mu_\alpha y^2] + [\mu_\delta\, y^2]} \quad \dots\dots (12)$$

2. The alternative solution is that $V_{\alpha\odot}$ and $V_{\delta\odot}$ are considered as unknown constants to be determined from the equations.

Next in the same way as explained in section 3, the stars are arranged in equal distance groups $y \pm \frac{1}{2} dy$. For each group we obtain a set of equations of the shape (11), but with $\overline{V}_{\alpha p} = \overline{V}_{\delta p} = 0$. Next the numerical values of $V_{\alpha\odot}$, $V_{\delta\odot}$ and a are obtained from a least squares solution. Just as with the solution for the radial velocities the numbers of stars which are available in each group will be small and so it is doubtful whether the condition $[V_{\alpha p}] = [V_{\delta p}] = 0$ is fulfilled. If not, serious errors in the computed values will occur. It is therefore preferable not to use this alternative.

6. Determination of *a* from stars distributed over a large area of the sky.

If the available stars are distributed over a large area of the sky, $V_{R\odot}$, $V_{\alpha\odot}$ and $V_{\delta\odot}$ can no longer be considered a constant. Now the space velocities of the stars along the three major axes X, Y and Z are

$$\left.\begin{aligned} \dot{x} &= -K\mu_\alpha r \sin\alpha - R\mu_\delta r \cos\alpha \sin\delta + V_R \cos\alpha \cos\delta \\ \dot{y} &= K\mu_\alpha r \cos\alpha - K\mu_\delta r \sin\alpha \sin\delta + V_R \sin\alpha \cos\delta \\ \dot{z} &= + K\mu_\delta r \cos\delta + V_R \sin\delta \end{aligned}\right\} \quad \ldots (13)$$

If in this equation we write in the same way as before

$$\dot{x} = -\dot{X}_\odot + \dot{x}_p;\ \dot{y} = -\dot{Y}_\odot + \dot{y}_p;\ \dot{z} = -\dot{Z}_\odot + \dot{z}_p;\ r = y(1+ay)$$

and $$V_R = V_{R,y} - ay^2 R(b, l)$$

and on the right hand side of the equations we collect all terms which do not contain a and also those containing a, we obtain a set of equations of the general form

$$\left.\begin{aligned} -\dot{X}_\odot + x_p &= A_1 + A_2 a \\ -\dot{Y}_\odot + y_p &= B_1 + B_2 a \\ -\dot{Z}_\odot + z_p &= C_1 + C_2 a \end{aligned}\right\} \quad \ldots . (14)$$

In these equations the coefficient $A_1, A_2 \cdots C_2$ can be computed from the observational data. As before two different solutions seem to be possible. With the first for $\dot{X}_{\odot}$, $\dot{Y}_{\odot}$ and $\dot{Z}_{\odot}$ the known components of the solar velocity are inserted. Next the equations (14) for the separate stars are summed.

In the summations we take $[\dot{x}_p] = [\dot{y}_p] = [\dot{z}_p] = 0$ and the numerical value of a can immediately be computed. Eventually the three sets of equations can be added to find the weighted mean value of a.

As before the second alternative is that $\dot{X}_{\odot}$, $\dot{Y}_{\odot}$ and $\dot{Z}_{\odot}$ are considered as unknown constants. The stars are grouped in aequidistance intervals and $\dot{X}_{\odot}$, $\dot{Y}_{\odot}$ and $\dot{Z}_{\odot}$ as well as a are found from a least squares solution. For the same reason as before this second alternative cannot be recommended.

The advantage of using the equation (13) would be that a fixed system of axes is used.

However, the computation of the component of space velocity for all individual stars involves a great amount of numerical work. The advantage of having a fixed system of axes is not large and therefore it will be preferable in the calculations to use the relations (7) and (12) to compute a. It may however be necessary to use the more elaborate equations (13) and (14) for certain groups of dwarf stars, for which the total numbers which appear in the catalogues are so small, that no subdivision into different areas is possible. But it is doubtful whether with these stars any results can be obtained.

7. The function $\alpha(y)$

Up till now it has been supposed that a is a constant. Also the solution of the equations (7) and (12) will give us the mean absorption coefficient $a(y)$ for all stars up to the apparent distance y. If $\alpha(y)$ is the true absorption coefficient at the distance y we evidently have

$$y\,\overline{a(y)} = \int_0^y a(y)\,dy \;ox\; \alpha(y) = \frac{d(y\,\overline{a(y)})}{dy}$$

For practical application it will be sufficient if we write

$$a(y) = \frac{\Delta \{ y \overline{a(y)} \}}{\Delta y} \quad \ldots. \; (15)$$

where we have to remember that of necessity $a(y)$ must be negative. Whether the results will be of sufficient accuracy to determine the function $a(y)$ remains to be seen.

The way to proceed would be to solve the equations (7) and (12) for all stars up to the distance $y_1, y_2 \cdots y_n$ and to plot the values $a(y)$ against the corresponding values y. If a smooth curve is drawn through these points according (15) the tangent to this curve is equal to the desired value.

On the whole, interstellar matter is thought to be much concentrated along the galactic plane.

If the interstellar medium is assumed to be stratified parallel to the galactic plane and to thin out gradually with increasing height z above or below the galactic plane, we may adopt as an approximation for the extinction coefficient a formula of the type

$$a = a_0 \, e^{\frac{-z^2}{2h^2}} \quad \ldots. \; (16)$$

where $z = r \sin b$

For the average extinction coefficient in the galactic plane near the sun usually a value of 1.0 magnitude per 1000 parsecs is adopted (photographic scale). From the study of extragalactic nebulae the total absorption perpendicular to the galactic plane is found to be approximately $0^m.25$. From these two values the numerical value of h in (16) has been found[3] to be $h = 200$ parsecs. Therefore, according to the adopted law (16) within a distance of 200 parsecs from the galactic plane the extinction coefficient diminishes from a_0 to $a_0 \, e^{-1/2} = 0.61 \, a_0$.

Therefore in the higher galactic longitudes there seems to be a reasonable possibility that some variations may be detected.

8. Results to be expected.

From the method proposed in the aforegoing sections, no immediate and spectacular results can be expected. In the first place all members of moving clusters must be excluded. The members of such moving clusters all have one and the same peculiar velocity in a particular direction and if such members were to be included $[V_p]$ cannot be expected to be zero. Also the high velocity stars of population II will have to be excluded while relatively to those stars the solar velocity is quite different from that relative to the population I stars. Quite generally the material must be homogeneous and consist of stars with a limited spectral range and of approximately one and the same absolute magnitude in order to avoid a systematic variation of $V_{\odot}$ with Y. But apart from these points no groups of stars must be considered for which nothing but spurious results can be expected. Generally these are the groups of dwarf stars for which all members which have been satisfactory observed are within a distance of 50 parsecs from the sun. In order to elaborate this point, the numbers in table 1 must be consulted. When drawing up this table, for the numerical value of a the mean overall absorption in the surroundings of the sun has been used viz. $a = 1$ magn, per kiloparsec. The second row of this table contains the excess of the apparent magnitude corresponding to the true distances r given in the first row. The third row gives the ratios y/r and the fourth the differences $y - r = y - (1 + ay)y = ay^2$. The maximum value which the correction term $y^2 R(l, b)$ in equation (5) and (7) can attain, is with $b = 0^\circ \pm n\pi$ and $(l - l_0) = 45^\circ \pm \frac{1}{2} n\pi$. The maximum values are given in the fifth row of the table. For the individual stars these maxima therefore also represent the maximum values $V_{R\odot} + V_{R,y}$ in equation (7). Consequently if the solution for a is going to have any weight, stars up to at least a distance of 500 parsecs will have to be included. Even then fairly large numbers of stars will have to be available.

If for the individual stars the variance of the peculiar velocities in one coordinate is taken to be 16 km/sec and 100 stars are observed, the error in $\bar{V}_p$ will be of the order $\pm$ 1.6 km/sec., that is to say of the same order as $V_{R\odot} + \frac{1}{N}[V_{R,y}]$ On the other hand with the distances $r > 500$ where the residuals $V_{R\odot} + V_{R,y}$ become larger, the approximation (3) may no longer be adequate. Eventually therefore low weights will have to be assigned to values of a derived in this way from radial velocities only.

TABLE I

(For explanation of the table see text (section 8))

1	r	100	200	300	400	500	600	700	800	900	1000	parsecs
2	y/r	1.047	1.096	1.148	1.202	1.259	1.318	1.380	1.445	1.514	1.505	
3	$y-r$	3	18	39	67	103	146	193	248	304	366	parsecs
4	$A(y-r)$	0.04	0.25	0.55	0.94	1.44	2.04	2.70	3.47	4.26	5.12	km/sec.
5	μ	0.042	0.021	0.014	0.011	0.008	0.007	0.006	0.005	0.005	0.004	seconds of arc
6	$V_y - V_R$	1	2	3	4	5	6	8	9	10	12	km/sec.
7	π	0.0100	0.0050	0.0033	0.0025	0.0020	0.0016	0.0014	0.0012	0.0011	0.0010	seconds of arc
8	π_{sp}	0.0096	0.0045	0.0029	0.0021	0.0016	0.0012	0.0010	0.0008	0.0007	0.0006	

We now turn our attention to the tangential velocities. We take the space velocity of the sun to be 20 km/sec. The corresponding value of the total proper motion μ is $20\,\pi/K$ seconds of arc.

The numerical values of μ corresponding to the various distances r and to this particular value of the solar velocity appear in the sixth row of the table. If for the converse operation, that is the determination of the solar velocity, instead of r the apparent distance y is used, the resulting velocity will be too large by the amount $V_y - V_r = (y - r)\,K\,\mu$.

The values $V_y - V_r$ appear in the seventh row of the table. Obviously these differences correspond to the individual values $(V_{\odot} + \mu y)$. From the table it is evident that with $r = 500$, the numerical values $(V_{\odot} + \mu y)$ are already considerable, so that it should be possible to obtain non trivial values of a.

If values of $r > 500$ are considered, the proper motions μ are so small that the probable error corresponds to a substantial fraction of the total proper motion, and non spurious results are obtainable only if large numbers of stars are available. Anyhow, it seems that if sufficient care is exercised, the proposed method may lead to good results for highly luminous stars at sufficient angular distance from the solar Apex and Anti Apex. Near the Apex and the Anti Apex all terms appearing on the righthand side of equation (12) must be zero and the result is indeterminate. Finally the question can be considered whether acceptable results can be expected when this method is applied to groups of stars for which the absolute magnitude can be derived from some physical parameter. With the Cepheids and the long periodic variables the period length can be considered as being such a parameter. With these groups of stars the available numbers, for which the radial velocity and the proper motions have been determined with a sufficient degree of accuracy, are small. For a large number of Cepheids the radial velocities are well determined and so it may be worth while to attempt applying the equations (5) and (7).

The Cepheid variables, though limited to galactic plane, are distributed over all longitudes and eventually it will be necessary to compute for each star the local value $V_{R\odot}$ instead of inserting a certain mean. At best it might be possible to ar-range the variables in order of their longitude, and separately consider the groups within (wide) longitude intervals. Eventually it would be necessary to study the influence of the second order term both in the development (3) and in the effect of differential galactic rotation.

With the long periodic variables good care will have to be taken that the group is homogeneous. That is to say that the period intervals must be strictly defined. Of the different subgroups of long periodic variables several are well represented in high galactic latitudes, so that a solution for different latitude may be attempted in order to see whether the coefficient of extinction (16) can be determined. If only radial velocities are used, the local components $V_{R\odot}$ will have to be computed. If for a sufficient number of stars the components of the proper motion are given, it may be an advantage if the equations (14) are used.

Obviously before solving for a the three equations should be added together. It should finally be remarked that before embarking on any attempt to apply this method the available material should be carefully scrutinised in order to ascertain that sufficient observational data are available. In many instances this will not be the case.

9. The use of the distribution of the peculiar velocities.

Wilh the methods discussed in the previous sections, the prerequisite is that $\frac{1}{N}[V_p] \rightarrow 0$. As the standard deviation $S = \frac{1}{N}\{V_1^2 + V_2^2 + \cdots V_n^2\}^{1/2}$ of the peculiar velocities is considerable, this will only be the case if large numbers of observations are available. The numbers of stars for which spectroscopic parallaxes or both spectroscopic parallaxes and radial velocities are known is fairly limited. Consequently in most instances the residual values $[V_p]$ which occur in the equations, have a probable numerical value which is of the same order of magnitude as the differences from which a is to be computed. The resulting solution for a will have small weight or no weight at all. It must therefore be of advantage to have a method which is independent from the residual values $[V_p]$.

The equations (6) and (11) can be written in a slightly different way.

$$-V_{\alpha\odot} + V_{\alpha p} = K\mu_\alpha y - \left(K\mu_\alpha y - K\mu_\alpha y\, e^{+ay}\right)$$ which easily

reduces to $$\frac{K\mu_\alpha y}{-V_{\alpha\odot} + V_{\alpha\beta}} = e^{-ay}$$

From this it immediately follows that also

$$\frac{Ky\,\sigma(\Delta\mu_\alpha)}{S(\alpha)} = e^{-ay}; \qquad \frac{Ky\,\sigma(\Delta\mu_\delta)}{S(\beta)} = e^{-ay}$$

and so

$$\frac{Ky\left\{\sigma(\Delta\mu_\alpha) + \sigma(\Delta\mu_\delta)\right\}}{S(\alpha) + S(\beta)} = e^{-ay} \qquad \dots\dots (17)$$

where

$$\sigma(\Delta\mu_\alpha) = \sqrt{\frac{[(\mu_\alpha - \bar{\mu}_\alpha)^2]}{n}} \quad \text{and} \quad \sigma(\Delta\mu_\delta) = \sqrt{\frac{[(\mu_\delta - \bar{\mu}_\delta)^2]}{n}}$$

Eventually instead of the standard deviation for σ and S the probable or the mean deviation of apparent motions and the space velocities may be taken. For the stars in a limited area of the sky and of one and the same spectral type $S(\alpha)$ and $S(\delta)$ are constants, corresponding to the lengths of the axes of the velocity ellipsoid in the directions determined by the points $\{\delta = 0;\ \alpha \pm 90^\circ\}$ and $\{\alpha;\ \delta \pm 90^\circ\}$ where α and δ are the right ascension and declination of the center of the area considered. For stars of different spectral types along the main sequence and along the giant series, the principal axes of the velocity ellipsoid have been determined by Nordström [4], Strömberg [5] and others. Therefore the numerical values of $S(\alpha)$ and $S(\delta)$ can be found.

If it is desired to combine the stars of different spectral types, the values $S(\alpha)$ and $S(\delta)$ are no longer constant but vary with spectral type. Therefore the observed values $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ must be reduced to a common scale. Let $S(\alpha)$ be the value of the standard error adopted for all spectral types together and $S_i(\alpha)$ that for the spectral type i.

$S(\alpha)$ can simply be taken equal to $S(\alpha) = \overline{S_i(\alpha)}$. Now put

$\Delta \mu'_\alpha = \frac{S_i(\alpha)}{S(\alpha)} \Delta \mu_\alpha$ and $\Delta \mu'_\delta = \mu_\delta \cdot S_i(\delta)/S(\delta)$. Then in the equation (17) instead of $\sigma(\Delta \mu_\alpha)$ and $\sigma(\Delta \mu_\delta)$ the values $\sigma(\Delta \mu'_\alpha)$ and $\sigma(\Delta \mu'_\delta)$ are to be inserted. $\sigma(\Delta \mu'_\alpha)$ is to be computed from $\sigma(\Delta \mu'_\alpha) = \left\{ \frac{1}{n} \left[(\Delta \mu'_\alpha)^2 \right] \right\}$ while a similar relation holds for $\sigma(\Delta \mu'_\delta)$.

For $S(\alpha)$ and $S(\delta)$ the adopted mean values must be used. It is evident that as long as only stars from a limited area of the sky are considered, all systematic errors in both coordinates are automatically eliminated from the differences $\Delta \mu_\alpha = \mu_\alpha - \overline{\mu}_\alpha$ and $\Delta \mu_\delta = \mu_\delta - \overline{\mu}_\delta$. It will even not be necessary to apply the corrections $\Delta_1 \mu_\alpha$, $\Delta_2 \mu_\alpha$, $\Delta_1 \mu \delta$ and $\Delta_2 \mu \delta$ of the equations (4) and (9).

It would therefore seem that the equation (17) offers a fairly rapid and accurate method for evaluating the variation of e^{-ay} with y. Before computing the numerical value a, the influence of probable error in the determination of the proper motions will have to be eliminated.

From the numbers in table I it is evident that no good results can be expected with stars for which $\pi_S = 0'',001$· Obviously this value stands for all spectroscopic parallaxes between the limits 0".0015 and 0".0005, which limits correspond to a range in distance from about 500 to 2000 parsecs.

Eventually such a value e^{-ay} could at best indicate the value of the absorption for some intermediate distance y. Moreover in the parallax catalogue the number of stars to which the parallax 0".001 has been assigned is small. Due to the small numbers of stars which are available it will also be difficult to obtain good values e^{-ay} for $\pi_S = 0''.002$ and $\pi_S = 0''003$. Judging from appearances it would seem that the best results will be obtained for stars with a range of π_S from about $\pi_S = 0''.003$ to $\pi_S = 0''.010$ the larger parallax value will be only of value to see whether actually $e^{-ay} \rightarrow 1$.

It will not be possible to combine in one set stars from widely different areas of the sky. In the first place it would of course be necessary to apply the corrections $\Delta_1 \mu_\alpha$, $\Delta_2 \mu$, $\Delta_1 \mu \delta$ and $\Delta_2 \mu \delta$. This would still be possible. But apart from this a correction for the parallactic motion would have to be applied.

The size of this correction would involve the use of $r = y\, e^{ay}$ and therefore of the quantity a which is still to be determined. In such a cıse the only possibility woul be, after applyirg the systematic corrections, to determine the tau component of proper motion. The corresponding value $S_i(\tau)$ can be found from the velocity ellipsoid. Next the values $\tau^i = S_i(\tau)/S(\tau)$ can be obtained. So the equation for e^{-ay} would be

$$\frac{K\, y\sigma(\tau^i)}{S(\tau)} = e^{-ay} \qquad \cdots\cdot \quad (18)$$

Eventually the determination of $\sigma(\tau^1)$ would involve a considerable amount af numerical work.

The only stars for which it might be worth while to consider this alternative are those with $\pi_S = 0''.001$ and $\pi_S = 0''.002$. But as explained before, for $\pi_S = 0'',001$ only a value e^{-ay} valid for some rather undefined mean distance y would be obtained. It finally remains to be considered whether the standard deviation could also be used with the radial velocities. The answer is negative.

Instead of (6) we may write

$$- V_{R\odot} + V_{Rp} = V_R\,(\text{obs}) - y\, R\,(b, l) + \{y\, R\,(b, l) - y\, e^{ay}\, R\,(b, l)\}$$

or

$$- V_{R\odot} + V_{Rp} = V_R\,(\text{obs}) - y\, R\,(b, l) + P\,(y).$$

If for different values of y the standard deviation of the radial velocities is computed from

$$\frac{1}{N}\,[V_{Rp}^2] = \frac{1}{N}\,[\{V_R\,(\text{osb}) + V_{R\odot} - y\, R\,(b, l)\}^2]$$

in each term the error $P(y)$ is introduced. If σ_R is the computed and σ'_R the correct value of the standard error σ_R is equal to $\sigma_R^2 = \sigma_R'^2 + P\,(y)^2$. Even with the B stars the standard deviation of the velocity along one coordinate axis is about 10 km/sec. From table 1 it can be seen that with $\pi_S = 0''.001$ the numerical value of $P(y)$ has only increased to about $P(y) = 3$ km/sec. Therefore σ' would be $\sigma' = \sqrt{100} =$ km/sec. and $\sigma = \sqrt{110} = 10.5$ km/sec. With the other spectral types the value of the true standard deviation is of the order of 20 Km/sec. and σ and σ' would be $\sigma' = 20$ and $\sigma = 20.2$ respectively. Even when large numbers of stars with $\pi_S = 0''.001$ were available, it would be impossible to establish the difference between σ and σ'.

10. Alternative use of the standard deviation.

By applying a slight change to the method discussed in the preceding section, the method of the standard variations may perhaps still be used both with the radial velocities and with the proper motions. The radial component of the stars' peculiar velocity being given by $V_{R_p} = V_R\,(\text{obs}) + V_{R\odot} - y\,R\,(b, l)$ the standard deviation of the peculiar velocities is given by

$$\sigma_R^2 = \frac{1}{N}\left[\left\{V_R\,(\text{obs}) + V_{R\odot} - y\,R\,(l, b) - a\,y^2\,R\,(l, b)\right\}^2\right] \quad \cdots\cdot(19)$$

If by neglecting or by misjudging the influence of absorption erroneous distances are assigned to the individual stars, this causes an increase in the numerical value of σ_R. This is because the stars at different distances are influenced in an unequal way. For σ_R the smallest numerical value is obtained if the correct distances are inserted. This leads to the condition $\partial\sigma_R/\partial a = 0$. It may be assumed that this minimum value of σ_R also is its optimum value. From the equation (19) and the condition $\partial\sigma_R/\partial a = 0$ the following expression for a is obtained.

$$a = \frac{[y^2 R\,(l, b)\,V_R] + [y^2 R\,(l, b)\,V_{R\odot}] - [y^3 R^2\,(l, b)]}{[y^4 R^2\,(l, b)]} \quad \cdots\cdot(20)$$

If this relation is applied to the stars in a limited area of the sky, that is an area within which both $V_{R\odot}$ and $R\,(l, b)$ may be taken to be a constant the relation (20) is reduced to a more convenient shape

$$a = \frac{1}{R\,(l, b)}\;\frac{[y^2 V_R] + [y^2]\,V_{R\odot} - [y^3]\,R\,(l, b)}{[y^4]} \quad \cdots\cdot(21)$$

If possible, the equation (21) should be applied, because its use does not imply too extensive numerical calculations. The condition for its usefulness is that sufficient observational material is available to allow a subdivision into small areas. When such is not the case the relation (20) will have to be used, but it is then to be observed that the values $R(l, b)$ and $V_{R\odot}$ vary from one star to the other.

The question may be raised whether the value which is to be computed from the relative space velocity of the sun has not already been slightly falsified by the influence of absorption. From the shape of the equation (19) it is evident that any error in the determination of $V_{R\odot}$ will also tend to increase the numerical value of the standard variation σ_R. The correct value of $V_{R\odot}$ could therefore be found from the condition $\partial\sigma_R/\partial V_{\odot}$ which when applied to the equation (19) leads to the relation

$$V_{\odot} = \{a\,[y^2 R(l, b) \cos \lambda_1] - [V_R \cos \lambda_1] + [y\, R(l, b) \cos \lambda_1]\} \times \frac{1}{[\cos^2\lambda_1]} \quad \cdots\cdots (22)$$

which for a restricted area of the sky reduces to

$$V_R = \frac{R(l, b)}{N} \left\{ a\,[y^2] - \frac{[V_R]}{R(l, b)} + [y] \right\} \quad \cdots\cdots (23)$$

Using these latter equations to eliminate $V_{R\odot}$ from (20) and (21) and collecting the term in a, it is found that a is given by either

$$a = \frac{[y^2 R(l, b)]\,[\cos^2 \lambda_1] - [y^2 R(l, b) \cos \lambda_1]\,[V_R \cos \lambda_1] + [y^2 R(l, b) \cos \lambda_1]\,[y\, R(l, b) \cos \lambda_1]}{[y^4 R^2(l, b)]\,[\cos^2 \lambda_1] - [y^2 R(l, b) \cos \lambda_1]\,[y^2 R(l, b) \cos \lambda_1]} \quad \cdots (24)$$

or

$$a = \frac{\left\{[y^2 V_R] - \frac{1}{N}[y^2]\,[V_R]\right\} - R(l, b)\left\{[y^3] - \frac{1}{N}[y^2]\,[y]\right\}}{R(l, b)\left\{[y^4] - \frac{1}{N}[y^2]\,[y^2]\right\}} \quad \cdots\cdots (25)$$

Here again the advantage of the equation (24) is that it can be applied to stars distributed over wide areas of the sky while (25) is valid for a limited part of the sky only. The equation (25) can be applied when for such a restricted area of the sky large numbers of stars are available. It is to be observed that with the method discusssed in this section, all stars are to be included into the calculations and therefore also the stars with large values π_S. Of these nearby stars large numbers occur in the parallax catalogue. From the equations (22), (23), (24) and (25) it appears that the value $[V_R]$ is mainly determined by the numerous nearby stars which are available, while the coefficient of absorption a is mainly determined by the large distance stars.

In the equations (24) and (25) no terms occur which explicitly contain the component of the solar velocity $V_{R\odot}$. However, from the way these equations were derived, it will be evident that actually the values of a found from these equations are not independent of solar velocity. Therefore the question should carefully be considered whether the value of a is to be computed from the equations (20) and (21) or from (24) and (25).

When the equations (20) and (21) are used, the value $V_{R\odot}$ to be inserted is based on large numbers of observations covering the whole sky. When using the equations (24) and (25) actually a value of the solar velocity is used based on observations which are locally restricted, but may have the advantage that local irregularities are eliminated. Probably the best procedure will be to make both solutions and compare the resnlts. This will give an impression about the reliability of the results obtained. Due to the fact that with both procedures the nearby stars are to be included, the uncertainty in the determination of the value $\frac{1}{N}[V_{\rho p}]$ seems to have been removed.

Once the numerical value of a has been computed from (24) or (25) this value a may be inserted in (22) to determine the local component of solar velocity.

A procedure similar to that applied to the radial velocities can be applied to the proper motion components μ_α and μ_δ. Only the equations for μ_α will be written out fully. Those for are obtained by simple substitution of the suffic δ for α. The peculiar velocity of a star in the α direction is equal to

$$V_{\alpha p} = Ky(\mu_\alpha + \Delta\mu_\alpha) + V_{\alpha\odot} + Ky^2 a(\mu_\alpha + \Delta\mu_\alpha)$$

where $\Delta\mu_\alpha$ stands for the sum of all corrections to be applied to the observed proper motion in the α directions The standard deviation σ_α is given by

$$\sigma_\alpha^2 = \frac{K}{N}\left[\left\{y(\mu_\alpha + \Delta\,\mu_\alpha) + V'_{\alpha\odot} + y^2 a(\mu_\alpha + \Delta\,\mu_\alpha)\right\}^2\right]$$

where $V'_{\alpha\odot} = \frac{V_{\alpha\odot}}{K}$. Introducing the conditions $\partial\sigma_\alpha/\partial a = 0$ and $\partial\sigma_\alpha/\partial V'_\odot = 0$ the following equations are obtained

$$a = -\frac{\{[y^3(\mu_\alpha + \Delta\,\mu_\alpha)^2] + V'_\odot[y^2(\mu_\alpha + \Delta\,\mu_\alpha)\cos\lambda_2)\}}{[y^4(\mu_\alpha + \Delta\,\mu_\alpha)^2]} \quad \cdots(26)$$

$$V'_\odot = -\frac{\{[y(\mu_\alpha + \Delta\,\mu_\alpha)\cos\lambda_2] + a\,[y^2(\mu_\alpha + \Delta\,\mu_\alpha)\cos\lambda_2]\}}{[\cos^2\lambda_2]} \quad \cdots(27)$$

and when $V'_\odot$ is eliminated between (26) and (27) the solution for a is found to be

$$a = \frac{[y(\mu_\alpha + \Delta\,\mu_\alpha)\cos\lambda_2]\,[y^2(\mu_\alpha + \Delta\,\mu_\alpha)\cos\lambda_2] - [y^3(\mu_\alpha + \Delta\,\mu_\alpha)^2]\,[\cos^2\lambda_2]}{[y^4(\mu_\alpha + \Delta\mu_\alpha)^2]\,[\cos^2\lambda_2] - [y^2(\mu_\alpha + \Delta\mu_\alpha)\cos\lambda_2]\,[y^2(\mu_\alpha + \mu_\alpha)\cos\lambda_2]} \quad \cdots(28)$$

If the available numbers of observations are so large that the relation can be applied to stars in limited areas of the sky, for each separate area $V'_{\alpha\odot} = V'_\odot\cos\lambda_2$ and $\Delta\mu_\alpha$ may be considered as a constant and the equations (26), (27) and (28) can be developed in powers of $\Delta\mu_\alpha$. The resulting formulae contain a rather large number of terms, but whenever they can be applied this will lead to a substantial decrease in the extent of the numerical work involved in the solutions. The relations will now be

$$a = -\frac{\{[y^3\mu_\alpha^2] + V'_{\alpha\odot}[\mu_\alpha y^2]\} + \{2\,[y^3\mu_\alpha] + V'_{\alpha\odot}[y^2)\}\,\Delta\mu_\alpha + [y^3]\,(\Delta\mu_\alpha)^2}{[y^4\mu_\alpha^2] + 2\,[y^4\mu_\alpha]\,\Delta\mu_\alpha + [y^4]\,(\Delta\mu_\alpha)^2} \quad \cdots(29)$$

$$V'_{\alpha\odot} = -\frac{1}{N}\left\{([y\mu_\alpha] + a\,[y^2\mu_\alpha]) + ([y] + a\,[y^2])\,\Delta\mu_\alpha\right\} \quad \cdots(30)$$

$$a = \frac{-[y^3\mu_{\alpha^2}] + \frac{1}{N}[y^2\mu_\alpha][y\mu_\alpha] + \{-[y^3\mu_\alpha] + [y^2\mu_\alpha][y] + [y^2][y\mu_\alpha]\}\,\Delta\mu_\alpha + [y^2]\,[y]\,[\Delta\mu_\alpha)^2}{[y^4\mu_\alpha^2] + 2[y^4\mu_\alpha]\,\Delta\mu_\alpha + [y^4](\Delta\mu_\alpha)^2 - \frac{1}{N}\{[\mu_\alpha y^2][y^2]\Delta\mu_\alpha\}^2} \quad \cdots(31)$$

Just as previously, the equations (26), (27) and (28) can be applied to stars scattered over large areas of the sky, while (29) and (31) are valid only for limited areas while the condition for their usefulness is that sufficient observational data are available.

Notwithstanding their relative complicated shape, the application of (29) and (31) will cause a substantial decrease of the numerical work. With the proper motions also the question will have to be considered whether the better results are to be obtained from the solution of (29) or from the solution of (31). Here again it will be preferable to perform both solutions and to compare the results. It should be remarked that before solving a from these equations, they are to be combined with the equations to be derived for the δ coordinate.

The orientation of the three major axes of the velocity ellipsoid is not constant relatively to the local axes R, A and D, so that the local values σ_R, σ_α and σ_δ are not entirely constant.

Therefore if such is compatible with the available observational material, even when using the equations valid for larger areas, it seems advisable to apply some restrictions to the areas which are considered.

11. Effect of zero point error in the scale of absolute magnitudes.

The zero point of the spectroscopically determined absolute magnitudes has been determined from stars with known trigonometric parallaxes. The apparent magnitudes of those stars have certainly been affected by interstellar absorption. If this effect of absorption is not eliminated, the result will be an error in the zero point of the magnitude scale. The value of the absolute magnitudes M will be overrated, their true values being $M - \Delta M$ Especially with the older determinations there is a danger that this absorption effect has not sufficiently been taken into account. Consequently if later the spectroscopic distances are computed from the relation $m - M = 5 \log y$ instead as from $m - (M - \Delta M) = 5 \log y_1$, all values y will be too small. However, for stars of all distances the ratio y_1/y will be a constant $y_1/y = f$.

The presence of such a zero point error is not hypothetical. In a subsequent paper, to be published in this same series, the relations developed in the sections 3 and 5 are applied to the members of the Perseus group. The numerical results prove that a substantial zero point error must be present, although for this special case it is not possible to determine its numerical value with sufficient accuracy.

In all instances, where the presence of a zero point error is suspected, in the equations instead of y the value fy must be inserted, f being an unknown constant. The equations (21) and (23) for a limited area of the sky now read:

$$R(l, b)\, a f^2 = \frac{[y^2 V_R] + [y^2]\, V_{R\odot} - [y^3]\, R(l, b) f}{[y^4]} \quad \dots (32)$$

$$V_{R\odot} = \frac{1}{N} \left\{ a f^2 R(l, b) [y^2] - [V_R] + f [y] R(l, b) \right\} \quad \dots (33)$$

It should not be attempted to solve all three unknown quantities from one and the same set of data. The best procedure will be for $V_{R\odot}$ to adopt the value derived from the radial velocities in general, especially the value which results from the stars in the four directions of the galactic center, the anticenter and the two directions perpendicular to these. The value $V_{R\odot}$ obtained from these stars ought to be free from any influence of $y_1\, R(b, l)\, a$ and f. If $V_{R\odot}$ is considered as a known quantity, the solution can proceed in the following way. With the help of the equation (32) the term $a f^2 R(l, b)$ is eliminated from (33). The resulting equation contains only the unknown quantity f and is linear in f. The solution is

$$f = \frac{\left\{ N - \frac{[y^2]\,[y^2]}{[y^4]} \right\} V_{R\odot} + \left\{ [V_R] - \frac{[V_R y^2][y^2]}{[y^4]} \right\}}{R(l, b) \left\{ [y] - \frac{[y^2][y^3]}{[y^4]} \right\}} \quad \dots (34)$$

From this equation it should be possible to determine the numerical value of f without too much trouble. Once the value of f has been calculated, that of a follows immediately from (32).

In a similar way for a limited area of the sky the proper motions μ_α and μ_δ can be treated. After introduction of the quantity f in the equations (26) and (27) and the necessary reductions these equations read :

$$f a^2 = -\frac{f\{[y^3(\mu_\alpha + \Delta\mu_\alpha)^2] + V'_{\alpha\odot}[(\mu_\alpha + \Delta\mu_\alpha)y^2]\}}{[y^4(\mu_\alpha + \Delta\mu_\alpha)^2)} \quad \ldots\ldots(35)$$

$$V'_{\alpha\odot} = -\frac{1}{N}\{f[y(\mu_\alpha + \Delta\mu_\alpha) + a f^2[y^2(\mu_\alpha + \Delta\mu_\alpha)]\} \quad \cdots(36)$$

The numerical value of $V'_{\alpha\odot}$ is to be calculated from the space velocity of the sun as derived from the observed radial velocities. Next the term $a f^2$ is eliminated from (36) and the result is an equation linear in f of which the solution is

$$f = V'_{\alpha\odot}\frac{\{N[(\mu_\alpha + \Delta\mu_\alpha)^2 y^4] - [(\mu_\alpha + \Delta\mu_\alpha)^2 y^2]\}}{\{[\mu_\alpha + \Delta\mu_\alpha)^2 y^3][y^2] - [(\mu_\alpha + \Delta\mu_\alpha)y][y^4(\mu_\alpha + \Delta\mu_\alpha)^2\}} \quad (37)$$

Once the numerical value of f has been calculated from (37) this value of f can be inserted in (35) which will yield the numerical value of a. Evidently the equations for the δ coordinate are obtained by substituting in the equations (35), (36) and (37) δ for α but further there is no difference.

Obviously when an error in the zero point is present it is not only necessay to rewrite the equations of section 10, but also many of those in the preceding sections. In all instances for y the value $f y$ must be substituted. The resulting equations will not explicitly been given here. Not only will it be easy for the reader eventually to write them out, but also considering the different circumstances it would appear that from the different possibilities considered in this paper, the methods described in the sections 10 and 11 seem to be the most promising ones.

12. Summary

1. In the present article from a purely theoretical point of view the question is considered whether the material which is available for the spectroscopic parallaxes can be used to evaluate the coefficients of absorption and extinction.

2. Whatever use is made of this material, results can only be obtained if data of sufficient homogeneity are used, so that there are no real variations of the true velocity component systematically dependent on the distance. This implies that only stars of a limited spectral range and of approximately the same absolute magnitude can be used.

3. A first possibility is considered in the sections (3) and (5). The method discussed there is fairly simple and direct but the validity of the equations is sharply restricted by the fact that for the different distances $[V_p] \neq 0$. Especially with the distant stars on which the determination of a mainly depends $[V_p]$ may have such large residual values as to invalidate the results.

4. A second possibility is considered in section (9), but here also the stars with different distance y have to be separately considered. This second possibility is essentially based on the determination of σ the standard deviation of the peculiar velocities. The value which is computed for σ will be effected by probable error. Especially when the available numbers of stars are small, as is the case wlth the larger values of y, this probable error affecting σ may be so large that the determination of a may be unreliable.

5. The best result will eventually be obtained if stars of all distances can be combined into one set, thus substantially decreasing all possible residual values. This is realised in the third possibility discussed in section 10.

6. When applying this third possibility there will be two alternatives. With the first as the component of the solar velocity the value is used computed from the given relative velocity of the sun. With the second alternative a locally determined value of the solar velocity is used, which unavoidably will have relatively low weight, but may have the advantage

that local irregularities are eliminated. Eventually it will be advisable to apply both alternatives and to compare the results.

7. There is a possibility that there is a zero point error in the scale of absolute magnitudes which have been determined from spectroscopic observations If this is suspected to be the case, the second alternative as mentioned under 6 should not be used, but the known value of the component of the solar space velocity should be used for determining the constant correction factor f which has to be applied to the spectroscopic distances y. If in limited areas of the sky sufficient numbers of stars are available, the equations derived in section 11 can be used. If the stars are scattered over large areas of the sky the original equations (21), (22), (26) and (27) of section 10 have to be used. The values of $V_\odot$ and $R(l, b)$ to be inserted are the local ones, while instead of y the value yf must be used. The further solution remains unchanged.

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