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POLITICAL MORAL HAZARD

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ABSTRACT

This dissertation is composed of three chapters on the subject of moral hazard in the social and political sphere.

The first chapter explores the effects of asymmetric information on the public control of politicians in a world where the politicians' pre–election promises are not credible. Politicians make decisions that affect social welfare. Some of these decisions are observable by the public and some are not. We study a model with identical politicians and a representative voter whose interests conflict with those of the politicians. The voter's decision to re–elect the politician depends on both her welfare and the observable decisions of the politician. In equilibrium, either socially optimal decisions are not taken by the politician or if taken, the politician would extract more rent in each period. In the latter case, politicians are also replaced more frequently. We show that a reform that would make the political system more transparent should be supported by the public and may also be welcomed by the politician.

The second chapter analyzes the relation between political accountability and the size of provinces. We study a model in which the signal to the electorate about the politician's

decision is noisier in more highly populated provinces. The model implies that both the equilibrium re–election probabilities of politicians and political accountability are lower in highly populated provinces. This prediction of the model is tested on panel data of nine municipal elections for 81 provinces of Turkey in the period 1963-2004. We find that province size is negatively associated with political accountability.

In the third chapter, we examine the effects of the introduction of restrictions on the statement of preferences in a two–sided matching model with incomplete information. The model is similar to the process used for college admissions in Turkey. Colleges have unanimous preferences – students with higher ranking in the national examinations are always preferred. We show that the introduction of the restrictions on statement of students' preferences can result in unstable matching between colleges and students.

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List of Abbreviations

MLRP	Monotone Likelihood Ratio Property
PDF	Probability Density Function
SPE	Subgame Perfect Equilibrium
SSPE	Stationary Subgame Perfect Equilibrium

Chapter 1: Transparency and Political Moral Hazard

1.1 Introduction

In electoral competition, politicians run on a platform and make campaign promises. In turn, the electorate votes for the candidate whose platform they prefer. Once elected, politicians behave as though they forget these campaign promises. In general, there are no legal remedies to enforce these promises. Even with remedies, elected politicians may alter them or even ignore them since they now hold power. In such an environment, the voters will not pay any attention to the platform that the politicians advocate, so the campaign promises are not credible.

An elected politician's interests may differ from those of the public. For example, the policies the politician puts forth may be contrary to what the public wants. In other instances, the politician may simply choose to enrich himself while in office and disregard the public completely.

Political moral hazard arises as a consequence of two features. First, the politician has the authority to apply his policies for a certain interval of time between the elections. He obtains that power from the public in the elections. Second, there is asymmetric information between the politician and the public. In many cases, people do not have access to all the decisions the politician makes.

Following of Barro (1973) and Ferejohn (1986), the next election can be used as a mechanism to control politicians. If electors vote by reflecting on the results of the politician's policies for the previous period, the politician may apply policies that satisfy the electorate and consequently not lose, although he may disagree with those policies.

In a complete information environment, Barro (1973) demonstrated that elections can be employed as a device to control politicians, and in equilibrium, politicians get some rent for having the authority to apply their policies. Ferejohn (1986) furthered this argument in the existence of asymmetric information between the politician and the electorate. The politician observes the realization of a random variable and acts accordingly. The electorate's utility depends on both the action of the politician and the realization of the random variable. The electorate observes only their utility, and in equilibrium, if it is more than a threshold value, they re–elect the politician. After the politician observes the realization of the random variable, he will act to stay in the office if, and only if, the realization of the random variable is in some interval. The politician obtains rent from both having the authority to apply his policy and having superior information to the electorate. Persson, Roland and Tabellini (1997) advanced the model of Ferejohn by fully endogenizing the utility of the politician from holding the office in the future. Banks and Sundaram (1993) discussed a model in which the politician acts before observing the realization of the random variable. They show the existence of equilibrium where the politician is re–elected if and only if the electorate's utility is greater than a cut–off level. Adsera, Boix and Payne (2003) provide empirical support for the claim, which states that the higher the informational asymmetries between the politician and the electorate, the higher the rent of the politician.

We develop here a principal agent model with homogenous politicians and a representative voter. In any period, there is an elected politician to choose a two–dimensional policy that affects the voter's welfare. The politician's policy consists of the tax policy and the investment policy. His payoff is the difference between the collected tax and the amount of investment. The voter can observe the tax policy, but she is not able to observe the investment policy that stochastically affects the income of the voter. The voter has a higher expected income if the politician invests more. The voter enjoys her net income, which is her income minus the tax that she pays. The model captures the conflict of interest between the politician and the voter. The politician wants to collect more tax and to invest less, whereas conversely, the voter wants him to collect less tax and to invest more.

The essential feature of the model is that it allows some of the politician's actions (tax policy) to be observable to the voter and some (investment policy) not to be observable. Hence, the voter uses a retrospective decision rule for re–election of the politician that depends not only on the results of the politician's unobservable policies, but also on his observable actions. This notion represents the interaction between the politicians and the electorate better than the case where the politician has a one–dimensional policy that is unobservable by the electorate.

We study two variants of the model. First, we consider that the voter is able to commit to a strategy concerning whether or not to re–elect the politician. In each period, the voter chooses a strategy depending on her information set at the end of the period. Then, she decides whether to re–elect the politician according to that strategy. Second, we consider that the voter is unable to commit to a strategy. The politician chooses the policy and at the end of each period the voter chooses whether to re–elect the politician.

If the voter is able to commit to a strategy, there are two types of equilibria in stationary strategies. Depending on the effect of the politician's unobserved actions to the welfare of the voter, the power of the politician (maximum tax that he can collect from the voter), and the politician's discount rate of future payoffs, one of these two types exists. In the first type of equilibrium, the politician does not invest, collects a low tax, and is never replaced. In the second type of equilibrium the politician invests, collects more tax compared to the first type, and is replaced with some probability.

We show that the politician may not make the socially optimal decisions in a non–transparent political system. Even if the politician makes the socially optimal decisions, he extracts more rent each period and is replaced more frequently. It is generally believed that an increase in the transparency of the political system makes the voter better off and the politician worse off. Therefore, the politician prefers the political system to be less transparent, and whereas the voter prefers it to be more transparent. Our results imply that an increase in the transparency of the political system increases the voter's payoff, but may have no effect in the politician's payoff. The reasoning for no change in the politician's payoff goes as follows. When the transparency increases, the politician extracts less rent each period while in the office, so his per–period payoff decreases. On the other hand, the politician's probability of being re–elected for the next period increases and consequently his expected payoff from the future periods increases. We show that in case of an increase in the transparency, the negative effect of the reduction of the politician's per–period payoff on his lifetime payoff may cancel the positive effect of the increase of the probability of being re–elected on his lifetime payoff. Therefore, a reform that would make the political system more transparent should be supported by the public and may also be welcomed by the politician.

If the voter is unable to commit to a strategy, then there exist multiple equilibria. Even in a transparent political system, the socially optimal decision is not supported in some of these equilibria. The best equilibria for the voter are the ones that have the same outcome with the case of the voter being able to commit.

The outline of the chapter is as follows. In section 1.2, we present the model. In section 1.3, we consider the case of the voter being able to commit to strategy. In section 1.4, we consider the case of the voter being unable to commit to a strategy. Section 1.5 presents a numerical example. And finally, section 1.6 concludes. Most of the proofs are relegated to the Appendix.

1.2 Model

There is one elected politician and one representative voter. The politician's investment increases the voter's expected income. The voter cannot observe whether the politician invests. Because of the informational asymmetry between the politician and the voter, the moral hazard problem arises.

The politicians and the voter are infinitely lived and the politician of the first period is elected at the end of period 0. Note that by saying "the politician," we mean the incumbent politician. In each period $t \geq 1$, the politician chooses a two–dimensional policy: the tax policy and the investment policy. The payoff of the politician in period t is given by $(\tau_t - I_t)$, where τ_t is the tax collected and I_t is the amount of investment. We assume that $\tau_t \in [0, \overline{\tau}]$ and $I_t \in \left\{0, \hat{I}\right\}$. The politician can collect any positive amount of tax less than or equal to $\overline{\tau}$, which is the maximum allowable tax level. His investment policy can be either "investment" or "no investment." If the politician decides to invest, then he invests at level $\hat{I} > 0$. If his decision is "no investment," then he does not invest anything. The politician discounts the future payoffs by a discount factor $\delta < 1$. In any period, if the politician loses an election, he is never re–elected and his outside payoff is zero. We also assume that when there is an election, there exists at least one previously not elected opponent for the politician. The politician and his opponents are identical in terms of ability and preferences.

In each period, the voter observes the tax policy of the politician, but she cannot observe the investment policy. At the end of each period, there is an election and the voter may re–elect the politician or his opponent for the next period. The payoff of the voter in period t is her net income, which is equal to $(y_t - \tau_t)$. $y_t \in [0, \overline{y}]$ is the income of the voter in period t, which is affected by the investment decision of the politician for that period. The voter has a higher expected income if the politician invests. Let $f(y_t|I_t)$ be the conditional probability density function and $F(y_t|I_t)$ be the conditional cumulative distribution function of the voter's income in period t.

We make two assumptions about the conditional probability density function. The first one is the following.

$$\left[\int y_t f\left(y_t|\hat{I}\right) dy_t - \int y_t df\left(y_t|0\right) dy_t\right] \geq \hat{I}$$

This assumption implies that the expected benefit of the voter under the investment decision is more than the cost of investment. Therefore, the investment decision of the politician is socially optimal in every period. The second assumption is that the densities have the strict monotone likelihood ratio property (strict MLRP).

$$\frac{f\left(y_t|\hat{I}\right)}{f\left(y_t|0\right)} \text{ is strictly increasing in } y_t.$$

As y_t increases, the likelihood of getting income level y_t if the politician invests relative to the likelihood if the politician does not invest is strictly increasing.

The model creates an obvious conflict of interests between the politician and the voter. The politician prefers to collect more tax and not to invest, whereas the voter prefers to pay less tax and that the politician invests.

The ability of the voter to commit to a strategy whether to re–elect the politician is crucial in this game. The game in which the voter is able to commit to a strategy is called "the commitment game." The game in which the voter is unable to commit to a strategy is called "the no commitment game."

The sequence of the events in the commitment game is as follows. In each period $t \geq 1$:

1. The voter chooses a strategy whether to re–elect the politician conditional on her information set before the election that will be held at the end of period t.

2. The politician chooses the tax policy and the investment policy.

3. The income of the voter is drawn from a distribution that depends on the investment policy of the politician.

4. The election is held and the voter decides whether to re–elect the politician according to her strategy.

The sequence of the events in the no commitment game is as follows. In each period $t \geq 1$:

1. The politician chooses the tax policy and the investment policy.

2. The income of the voter is drawn from a distribution that depends on the investment policy of the politician.

3. Election is held and the voter decides whether to re–elect the politician.

Our equilibrium concept is the subgame perfect equilibrium in pure strategies. The strategies of the players are as follows. Let H^t denote the set of all possible histories through period t. In period t, the politician's strategy $\rho_t = (\tau_t, I_t) : H^{t-1} \to [0, \overline{\tau}] \times [0, \overline{y}]$ maps the set of all possible histories prior to period t into a policy. In period t, the voter's strategy $\varkappa_t : H^{t-1} \times [0, \overline{\tau}] \times [0, \overline{y}] \to \{0, 1\}$ maps the set of all possible histories prior to period t, the tax policy of the politician, and the income level into a decision: the voter re–elects the politician (1) or removes the politician from the office by electing his opponent(0).

The commitment game is discussed in the next section and the no commitment game is discussed in section 1.4.

1.3 The Commitment Game

The voter is able to commit to a strategy each period. We restrict our attention to the stationary subgame perfect equilibria (SSPE). This requires that players act identically and optimally when faced with identical continuation games, and hence imply history–independent strategies. In period t, the voter's strategy x_t can only depend on the (τ_t, y_t).

Definition 1 *The voter's stationary strategy is defined to be* monotone *if it is such that if the voter re–elects the politician in any period when the realized income is y and collected* *tax is τ for any $\tau \in [0, \overline{\tau}]$, then she re–elects the politician in that period also for any realization of income $y' > y$ when the politician collects the tax τ.*

In a monotone stationary strategy, in every period the voter re–elects the politician if and only if her income is higher than some threshold level for a given tax level.

Proposition 1 *Any SSPE outcome can be generated by an SSPE with monotone strategies.*

The proposition implies that in any SSPE, either the voter's strategy is monotone or there exists another SSPE with monotone strategies, and both the politician's policy and the re–election decision for the politician are identical in these two equilibria. Therefore, we can derive all the outcomes of SSPE by focusing solely on SSPE with monotone strategies.

Formally, any monotone stationary strategy of the voter can be represented by using a function $g : [0, \overline{\tau}] \to [0, \overline{y}]$ as follows.

$$\varkappa : \text{“}\varkappa = 1 \text{ if and only if } y \geq g(\tau) \text{ in any period”}$$

The threshold level of income in strategy $\varkappa$ is $g(\tau)$. For any arbitrary tax level τ, in any period, the politician is re–elected if and only if the realized income is greater than or equal to $g(\tau)$ in the case of the politician collecting the tax τ in that period.

The politician's stationary strategy ρ is choosing a policy (τ, I) in every period. Let $\alpha(\rho|\varkappa)$ be the probability of being re–elected for the politician when his strategy is ρ and the voter's strategy is $\varkappa$.

$$\begin{aligned} \alpha(\rho|\varkappa) &= \int_{g(\tau)}^{\overline{y}} f(y|I)\, dy \\ &= 1 - F(g(\tau)|I) \end{aligned} \tag{1}$$

$\alpha(\rho|\varkappa)$ is simply the probability of having the income greater than or equal to the threshold value in the voter's strategy. Let $U(\rho|\varkappa)$ denote the expected payoff of the politician when he chooses the strategy ρ given the voter's strategy is $\varkappa$.

Lemma 1 *Given the voter's strategy $\varkappa$, the politician's expected payoff from choosing the strategy ρ is as follows.*

$$U(\rho|\varkappa) = \frac{\tau - I}{1 - \delta\left(1 - F\left(g(\tau)|I\right)\right)}$$

The politician's payoff is composed of two parts: The current period's payoff and the discounted value of the expected future payoff. His payoff is increasing in the current period's payoff and in his probability of re–election. The voter's strategy does not affect the current period's payoff of the politician, but does affect the politician's re–election probability. The electoral control of the politician is based on this payoff structure. The voter's strategy can provide incentives for the politician to lower his current period's payoff in order to raise his probability of re–election.

Note that for any strategy of the voter, the politician can always have a minimum payoff $\overline{\tau}$ by collecting the maximum tax and not investing in each period. Let ρ_0 represent this strategy of the politician.

$$\rho_0 = (\overline{\tau}, 0)$$

Since the politician's payoff is at least $\overline{\tau}$ when he chooses ρ_0, in any SSPE, the payoff of the politician can not be less than $\overline{\tau}$.

The following proposition states a necessary condition for the existence of an SSPE in which the politician invests.

Proposition 2 *If* $\overline{\tau} < \frac{\hat{I}}{\delta}$*, then there exists no SSPE with investment.*

If the cost of investment is close to the maximum allowable tax, then for any strategy of the voter, the increase in the politician's expected future payoff because of the investment is less than the cost of investment. Thus, the politician does not invest. The maximum allowable tax level can be interpreted as the power of the politician. Hence, this result reveals that even if the expected benefit of the investment is much more than the cost of it, if the politician does not have sufficient power, then he does not make socially optimal decisions.

Another implication of the proposition is on the discount rate of the politician. If the politician has a high discount rate (low δ), then the voter is not able to give enough incentive to the politician to invest. Since the politician heavily discounts the future payoffs, the cost of investment is larger than the positive effect of the investment on his expected payoff. Consequently, the politician does not invest. For the remainder of this chapter, we assume that $\overline{\tau} \geq \frac{\hat{I}}{\delta}$.

To address the effects of the asymmetric information between the politician and the voter, we first consider the case where the voter observes both the tax policy and the investment policy of the politician in every period. Then, we consider the setup in the model in which the voter observes only the tax policy of the politician in every period. The case in which the voter observes both the tax policy and the investment policy of the politician in every period is called "the case of a transparent political system," while the case in which the voter observes only the tax policy of the politician in every period is called "the case of a non–transparent political system."

1.3.1 The Case of a Transparent Political System

The voter observes both the tax policy and the investment policy of the politician in every period. Hence, there is no information asymmetry between the politician and the voter. We assume that even though the investment policy is observable, it is not verifiable. That is, there are no legal remedies to enforce the investment when the politician chooses some particular tax policies. The voter can, at most, remove the politician from the office in the next election in order to penalize him. In the case of a transparent political system, the voter's strategy depends also on the investment policy of the politician.

We first find the lowest tax level that the voter can induce the politician to collect in the case of no investment. Then, we find the lowest tax level that the voter can induce the politician to collect and also to invest. The difference between these two tax levels is the cost of inducing the politician to invest. If the expected benefit of the investment is more than the cost of inducing the politician to invest, then in equilibrium the voter induces the politician to invest.

Lemma 2 *There does not exist an SSPE without investment where the tax is less than* $(1-\delta)\,\overline{\tau}$.

When the politician does not invest, the lowest tax that the voter can induce him to collect is $(1-\delta)\,\overline{\tau}$. The voter can achieve it by choosing the strategy "re–elect the politician if and only if he collects the tax $(1-\delta)\,\overline{\tau}$ in every period." The politician's best response to this strategy is to collect the tax $(1-\delta)\,\overline{\tau}$ and not to invest. This is what Persson, Roland and Tabellini (1997) call "the rents from power." Even in a transparent political system, since the politician has the authority to choose his policies, he gets a payoff that is more than his outside payoff.

Lemma 3 *There does not exist an SSPE with investment where the tax is less than* $\hat{I} + (1-\delta)\,\overline{\tau}$.

$\hat{I} + (1-\delta)\,\overline{\tau}$ is the lowest tax that the voter can induce the politician to collect and also to invest. The voter can achieve this by choosing the strategy "re–elect the politician if and only if he invests and collects the tax $\hat{I} + (1-\delta)\,\overline{\tau}$ in every period." The politician's best response to this strategy is to invest and to collect the tax $\hat{I} + (1-\delta)\,\overline{\tau}$. In the case of transparent political system, the cost of inducing the politician to invest is equal to only the cost of investment. The politician's payoff is the same whether the voter induces him to invest.

Proposition 3 *When the politician's policy is observable by the voter, there exist SSPE of the commitment game with unique outcome. In these equilibria, in every period the politician invests, collects the tax* $\hat{I} + (1-\delta)\,\overline{\tau}$*, and is never replaced.*

The cost of inducing the politician to invest is equal to the cost of investment. Also, it is assumed that the expected benefit of the investment is greater than the cost of it. Thus, the expected benefit of the investment is more than the cost of inducing the politician to invest. Consequently, the voter induces the politician to invest in every period. In SSPE, the voter can choose the strategy "re–elect the politician if and only if he invests and collects the tax $\hat{I} + (1-\delta)\,\overline{\tau}$ in every period." In addition to this strategy, there are infinitely many monotone and non–monotone strategies of the voter that constitute SSPE with the same outcome. In these strategies, the politician is certainly re–elected if he invests and collects the tax $\hat{I}+(1-\delta)\,\overline{\tau}$, and if the politician collects another tax level, then his expected payoff is less than or equal to $\overline{\tau}$.

Note that in equilibrium, the re–election decision of the politician does not depend on the income – the politician is never replaced. Since the voter observes the policy of the politician, she is able to assure the re–election of the politician in case of he is investing. Hence, she is able to decrease the cost of inducing the politician to invest as low as the cost of investment. Consequently, in equilibrium, the politician invests every period and thus makes the social optimal decision.

1.3.2 The Case of a Non–Transparent Political System

The voter does not observe the investment policy of the politician in any period. Hence, the voter's strategy $\varkappa$ cannot depend on the investment policy of the politician. In any SSPE, the voter's strategy $\varkappa$ solves the following.

$$\begin{aligned} & \max_{\{g(\tau)\}} \int y dF\,(y|I) - \tau \\ \text{subject to:} \qquad & (\tau, I) \in \operatorname*{arg\,max}_{\left\{\tau\in[0,\overline{\tau}],I\in\left\{0,\hat{I}\right\}\right\}} U\,(\tau, I|\varkappa) \end{aligned}$$

In any SSPE, the voter chooses the strategy such that the optimal policy of the politician under that strategy maximizes her expected payoff. The previous subsection shows that in a transparent political system, in equilibrium the politician invests and the voter's strategy does not depend on the income. However, in a non–transparent political system, if the voter's strategy does not depend on the income, then the politician will not invest. The reasoning for this is as follows. Investment is costly for the politician. If the voter's strategy is independent of the income, then the re–election probability of the politician will be the same whether or not he invests. The politician, then, does not have any incentive to invest. Therefore, in any period, in any SSPE with investment, the voter's strategy must depend on the income of that period. Accordingly, the politician is replaced with some probability in any SSPE with investment.

Lemma 4 *There does not exist an SSPE without investment where the tax is less than* $(1-\delta)\,\overline{\tau}$.

Obviously, the ability to observe the politician's investment policy does not affect the equilibrium in which the politician does not invest. As in the case of transparent political system, in the case in which politician does not invest, the lowest tax that the voter can induce the politician to collect is $(1-\delta)\,\overline{\tau}$. The voter can achieve this by selecting the strategy "re–elect the politician if and only if he collects the tax $(1-\delta)\,\overline{\tau}$ in every period." The politician's best response to this strategy is to collect the tax $(1-\delta)\,\overline{\tau}$ and not to invest.

To characterize the SSPE, we first determine whether the voter is able to induce the politician to invest. If she is not able to do so, then there is no SSPE with investment. Consequently, the voter chooses the strategy "re–elect the politician if and only if he collects the tax $(1-\delta)\,\overline{\tau}$ in every period" in order to induce the politician to collect the lowest tax. The politician's best response to that strategy is to collect the tax $(1-\delta)\,\overline{\tau}$ and not to invest. If the voter is able to induce the politician to invest, then we find the cost of inducing the politician to invest. The voter induces the politician to invest, and consequently there exist SSPE with investment if and only if the expected benefit of the investment is more than the cost of inducing the politician to invest.

Here, we define two functions ϕ and ψ. First, $\phi : [0, \overline{\tau}] \to [0, \overline{y}]$ is defined as follows.

$$\phi(\tau) = \begin{cases} F^{-1}\left(\frac{\tau - \hat{I} - (1-\delta)\overline{\tau}}{\delta \overline{\tau}} | \hat{I}\right) & \text{if } \tau \geq \hat{I} + (1-\delta)\overline{\tau} \\ 0 & \text{otherwise} \end{cases}$$

If the voter's strategy $\varkappa$ is such that $g(\tau) = \phi(\tau)$ for $\forall \tau \geq \hat{I} + (1-\delta)\overline{\tau}$, then the politician's expected payoff is equal to $\overline{\tau}$ when he chooses the strategy $\left(\tau, \hat{I}\right)$ for any tax $\tau \geq \hat{I} + (1-\delta)\overline{\tau}$.

Lemma 5 *Given the voter's strategy $\varkappa$, for any tax $\tau \geq \hat{I} + (1-\delta)\overline{\tau}$, if $g(\tau) > \phi(\tau)$, then the politician prefers ρ_0 to the strategy $\left(\tau, \hat{I}\right)$.*

The second function we define is ψ. Let $\psi : (0, \overline{y}) \to \mathbb{R}$ be the following.

$$\psi(y) = \frac{\hat{I}}{\delta} \frac{(1 - \delta + \delta F(y|0))}{F(y|0) - F\left(y|\hat{I}\right)}$$

$\psi(y)$ can be interpreted as follows. Given the voter's strategy $\varkappa$, the politician prefers the strategy $\left(\tau, \hat{I}\right)$ to the strategy $(\tau, 0)$ if and only if $\tau \geq \psi(g(\tau))$ for $\forall \tau \in [0, \overline{\tau}]$.

Note that $\lim\limits_{y \to 0} \psi(y) = +\infty$ and $\lim\limits_{y \to \overline{y}} \psi(y) = +\infty$.

Lemma 6 *$\psi(y)$ has a unique minimum point.*

Let y^* be the unique minimum point of $\psi(y)$ and $\tau^* = \psi(y^*)$. Since τ^* is the minimum value of $\psi(y)$, for any strategy of the voter, the politician prefers the strategy $(\tau, 0)$ to the strategy $\left(\tau, \hat{I}\right)$ for $\forall \tau < \tau^*$. Therefore, there is no strategy of the voter that induces the politician to invest and to collect a tax less than τ^*.

Lemma 7 *If $\overline{\tau} < \tau^*$, then there does not exist an SSPE with investment.*

Since there is no strategy of the voter that induces the politician to invest and to collect a tax less than τ^*, if the maximum allowable tax level, $\overline{\tau}$, is less than τ^*, then the the politician does not invest for any strategy of the voter. Consequently, there does not exist any SSPE with investment.

If $\tau^* \leq \overline{\tau}$, then the voter is able to induce the politician to invest. The lowest cost of it depends on the $\phi(\tau^*)$ and y^*.

If $\phi(\tau^*) \geq y^*$, then the politician prefers the strategy $\left(\tau^*, \hat{I}\right)$ to both $(\tau^*, 0)$ and ρ_0 under the voter's strategy "re–elect the politician if and only if the collected tax is τ^* and the income $y \geq y^*$ in any period."

Lemma 8 *If the voter is able to induce the politician to invest and $\phi(\tau^*) \geq y^*$, then τ^* is the lowest tax that the voter can induce the politician to collect and also to invest.*

If $\phi(\tau^*) < y^*$, then the politician prefers the strategy $\left(\tau^*, \hat{I}\right)$ to the strategy $(\tau^*, 0)$, but his payoff when he chooses the strategy $\left(\tau^*, \hat{I}\right)$ is less than $\overline{\tau}$. Therefore, he prefers ρ_0 to the strategy $\left(\tau^*, \hat{I}\right)$. In this case, the voter cannot induce the politician to invest and to collect only tax τ^*.

First note that $\lim_{y \to 0} \psi(y) = +\infty$ and $\psi(y)$ has a unique minimum point at y^*. Hence, $\psi(y)$ is decreasing in the interval $(0, y^*]$. Second, by definition of $\phi(\tau)$, it is an increasing function. Since these two conditions imply that if $\phi(\tau^*) < y^*$, there exists a unique tax level, say τ^{**}, which satisfies the following requirement.

$$\tau^{**} = \psi(\phi(\tau^{**})) \text{ and } \phi(\tau^{**}) < y^* \tag{2}$$

The politician prefers the strategy $\left(\tau^{**}, \hat{I}\right)$ to both the strategy $(\tau^{**}, 0)$ and ρ_0 under the voter's strategy "re–elect the politician if and only if the collected tax is τ^{**} and the income $y \geq \phi(\tau^{**})$ in any period."

Lemma 9 *If the voter is able to induce the politician to invest and $\phi(\tau^*) < y^*$, then τ^{**} is the lowest tax that the voter can induce the politician to collect and also to invest.*

When the voter is able to induce the politician to invest, she does so if the expected benefit of the investment is greater than its cost to the voter. Lemma 4 and lemma 8 imply that if $\phi(\tau^*) \geq y^*$, then the lowest cost of inducing the politician to invest is $\tau^* - (1-\delta)\overline{\tau}$. Furthermore, lemma 4 and lemma 9 imply that if $\phi(\tau^*) < y^*$, then the lowest cost of inducing the politician to invest is $\tau^{**} - (1-\delta)\overline{\tau}$. Therefore, the investment is *worthwhile* if and only if the following condition holds.

$$\left[\int yf\left(y|\hat{I}\right)dy - \int ydf\left(y|0\right)dy\right] \geq \begin{cases} \tau^* - (1-\delta)\overline{\tau} & \text{if } \phi(\tau^*) \geq y^* \\ \tau^{**} - (1-\delta)\overline{\tau} & \text{if } \phi(\tau^*) < y^* \end{cases}$$

The equilibria of the commitment game can be described as follows.

Proposition 4 *When the voter cannot observe the investment policy of the politician, there exist SSPE of the commitment game with unique outcome. If the voter is not able to induce the politician to invest or the investment is not worthwhile, then in equilibrium, the politician does not invest, collects the tax $(1-\delta)\overline{\tau}$, and is never replaced. If the voter is able to induce the politician to invest, the investment is worthwhile and $\phi(\tau^*) \geq y^*$, then in equilibrium, the politician invests, collects the tax τ^*, and is replaced with positive probability. If the voter is able to induce the politician to invest, the investment is worthwhile and $\phi(\tau^*) < y^*$, then in equilibrium, the politician invests, collects the tax τ^{**}, and is replaced with positive probability.*

In a non–transparent political system, the politician may not make the socially optimal decisions as a consequence of two factors. First, the politician may be unable to collect the sufficient tax that gives him the incentive to invest. Second, the cost of inducing the politician to invest through taxation is so high that it overrides the returns of the investment for the voter. Consequently, the voter may prefer to induce the politician to choose a policy with a lower tax and no investment instead of a policy with investment, even though the investment decision of the politician is socially optimal.

In equilibrium, the voter's strategy can be as follows. If she is not able to induce the politician to invest or the investment is not worthwhile, then her equilibrium strategy can be "re–elect the politician if and only if the collected tax is $(1-\delta)\overline{\tau}$." If the voter is able to induce the politician to invest, the investment is worthwhile and $\phi(\tau^*) \geq y^*$, then it can be "re–elect the politician if and only if the collected tax is τ^* and the income is greater than or equal to y^*." If the voter is able to induce the politician to invest, the investment is worthwhile and $\phi(\tau^*) < y^*$, then it can be "re–elect the politician if and only if the collected tax is τ^{**} and the income is greater than or equal to $\phi(\tau^{**})$."

Note that there are infinitely many monotone and non–monotone strategies that can be the voter's strategy in an SSPE with the same outcome. In these strategies, if the politician chooses the policy in Proposition 4, then the re–election decision of the politician is the same with the strategy described above. Moreover, if the politician selects another policy, then his expected payoff will be less than or equal to that when he chooses the policy in Proposition 4. Thus, the politician does not select another policy in any SSPE.

1.3.3 Welfare Analysis

Here, we analyze the effects of the presence of asymmetric information to both the welfare of the voter and the politician. In a transparent political system, the politician invests, collects the tax $(1-\delta)\overline{\tau}$, and is never replaced. Consequently, his payoff is equal to $\overline{\tau}$. In the case of non–transparent political system, in SSPE, the policy of the politician depends on the maximum allowable tax level, the effect of investment to the income of the voter, and the politician's discount rate for future payoffs.

Proposition 5 *The welfare effects of the asymmetric information can be described as follows.*

i. If the politician does not invest in SSPE, then asymmetric information causes an expected loss of $\int yf\left(y|\hat{I}\right)dy - \int ydf\left(y|0\right)dy - \hat{I}$ *in the voter's payoff in each period. The politician's per–period and expected payoffs are equal to those in the case of a transparent political system.*

ii. If the politician invests in SSPE and $\phi\left(\tau^*\right) < y^*$*, then the politician collects the tax* τ^{**}*. Asymmetric information causes a transfer of* $\tau^{**} - \left[\hat{I} + (1-\delta)\overline{\tau}\right]$ *from the voter to the politician in each period. Thus, the politician's per–period payoff is greater than that in the case of a transparent political system. However, his expected payoff is equal to that in the case of a transparent political system.*

iii. If the politician invests in SSPE and $\phi\left(\tau^*\right) \geq y^*$*, then the politician collects the tax* τ^**. Asymmetric information causes a transfer of* $\tau^* - \left[\hat{I} + (1-\delta)\overline{\tau}\right]$ *from the voter to the politician in each period. The politician's both per–period payoff and expected payoff are greater than those in the case of transparent political system.*

Proposition 5 implies the existence of three possible types of welfare effect of the asymmetric information. Depending of the dynamics of the society (the power of the politician, the influence of the politician's policies to the welfare of the society, the politician's discount rate for future payoffs), the society faces one of these types.

The most severe effect the asymmetric information occurs in the first type: The politician does not make the socially optimal decision and the voter cannot get any benefit from it. The politician is indifferent to having a transparent or non–transparent political system.

In the second type, the socially optimal decision is made by the politician, but the voter transfers some of its benefits to the politician in order to induce him to make it. The politician's per–period payoff is greater than that in a transparent political system due to the transfer from the voter. However, now the politician is replaced with positive probability. We show that the negative effect of the reduction of the politician's re–election probability to his expected payoff cancels the positive effect of the increase of the politician's per–period payoff to his expected payoff. Therefore, the politician's expected payoff is the same in a transparent or non–transparent political system.

If the society faces one of these two types of welfare effect, then the voter suffers from the existence of asymmetrical information but the politician is indifferent to having a transparent or non–transparent political system. Consequently, a reform intended to make the political system more transparent should be supported by the voter and should also be welcomed by the politician.

The third type of welfare effect of the asymmetric information is as follows. As in the second type, the politician makes the socially optimal decisions and the voter transfers some of its benefits to induce the politician to make it. However, here the positive effect of the increase of the politician's per–period payoff to his expected payoff is greater than the negative effect of the reduction of the politician's re–election probability to his expected payoff. Consequently, the politician prefers a non–transparent political system to a transparent political one.

1.4 The No Commitment Game

The voter is unable to commit to a strategy. In every period, first the politician chooses his policy, then the income of the voter is realized, and finally the voter decides whether or not to re–elect the politician. Note that since the politician and his opponents are identical in terms of ability and preferences, given the politician's strategy, the voter is indifferent to re–electing the politician and replacing him with his opponent. Thus, any strategy of the voter is weakly optimal. Therefore, the voter's strategy to remove the politician in the case of poor income realization or a high tax policy is not a credible threat to the politician.

First, we analyze the case of a transparent political system. Then, we analyze the case of a non–transparent political system. We restrict our attention to the stationary subgame perfect equilibria.

1.4.1 The Case of a Transparent Political System

The voter observes both the tax policy and the investment policy of the politician in every period. The voter's strategy depends also on the investment policy of the politician. We assume that even though the investment policy is observable, it is not verifiable. That is, there are no legal remedies to enforce the investment when the politician chooses some particular tax policies.

Lemma 10 *The lowest tax that can be supported in an SSPE without investment is* $(1-\delta)\overline{\tau}$ *and such an SSPE exists.*

Proof: For any strategy of the voter and any tax $\tau < (1-\delta)\overline{\tau}$, since

$$U((\tau, 0)|\varkappa) = \frac{\tau}{1-\delta(1-F(g(\tau)|I))} < \overline{\tau},$$

the politician prefers to choose ρ_0 instead of choosing a strategy with a tax level less than $(1-\delta)\overline{\tau}$. Therefore, any tax $\tau < (1-\delta)\overline{\tau}$ cannot be supported in SSPE without investment.

Also, the following strategies constitute an SSPE. In every period, the politician does not invest and collects the tax $(1-\delta)\overline{\tau}$ and the voter re–elects the politician if and only if he collects a tax less than or equal to $(1-\delta)\overline{\tau}$. $\square$

Lemma 11 *The lowest tax that can be supported in an SSPE with investment is* $\hat{I} + (1-\delta)\overline{\tau}$ *and such an SSPE exists.*

Proof: For any strategy of the voter and any tax $\tau < \hat{I} + (1-\delta)\overline{\tau}$, since

$$U\left(\left(\tau, \hat{I}\right)|\varkappa\right) < \overline{\tau},$$

the politician prefers to choose ρ_0 instead of choosing a strategy with investment and a tax level less than $\hat{I} + (1-\delta)\overline{\tau}$. Therefore, any tax $\tau < \hat{I} + (1-\delta)\overline{\tau}$ cannot be supported in SSPE with investment.

Also, the following strategies constitute an SSPE. In every period, the politician invests and collects the tax $\hat{I} + (1 - \delta)\,\overline{\tau}$ and the voter re–elects the politician if and only if he invests and collects a tax less than or equal to $\hat{I} + (1 - \delta)\,\overline{\tau}$. $\square$

Proposition 6 *When the politician's policy is observable by the voter, the no commitment game has multiple SSPE. In these equilibria, either the politician does not invest and collects a tax $\tau \geq (1 - \delta)\,\overline{\tau}$, or the politician invests and collects a tax $\tau \geq \hat{I} + (1 - \delta)\,\overline{\tau}$.*

Proof: Lemma 10 demonstrates that there does not exist an SSPE without investment in which the politician collects a tax less than $(1 - \delta)\,\overline{\tau}$. Moreover, any tax $\tau \geq (1 - \delta)\,\overline{\tau}$ can be supported in an SSPE without investment with the following strategies of the players: In every period, the politician does not invest and collects the tax τ and the voter re–elects the politician if and only if he collects a tax less than or equal to τ.

Lemma 11 also shows that there does not exist an SSPE with investment in which the politician collects a tax less than $\hat{I} + (1 - \delta)\,\overline{\tau}$. Moreover, any tax $\tau \geq \hat{I} + (1 - \delta)\,\overline{\tau}$ can be supported in an SSPE with investment with the following strategies of the players: In every period, the politician invests and collects the tax τ and the voter re–elects the politician if and only if he invests and collects a tax less than or equal to τ. $\square$

In a transparent political system, if the voter is able to commit, proposition 3 shows that there exist SSPE with a unique outcome in which the politician invests, collects the tax $\hat{I} + (1 - \delta)\,\overline{\tau}$, and is re–elected. However, if the voter is unable to commit, then her strategy to remove the politician from office in the case where he does not invest or collects a tax greater than $\hat{I} + (1 - \delta)\,\overline{\tau}$ is not a credible threat to the politician. Consequently, there are many equilibria as shown in Proposition 6. In some of these, the politician does not invest. From the voter's perspective, the best of those equilibria are the those that have the same outcome with the equilibria of the commitment game.

1.4.2 The Case of a Non–Transparent Political System

In this section, we derive the equilibria of the no commitment game when the investment policy of the politician is not observable to the politician.

Lemma 12 *The lowest tax that can be supported in SSPE without investment is* $(1-\delta)\,\overline{\tau}$ *and such an SSPE exists.*

Proof: For any strategy of the voter, since the politician prefers ρ_0 to any strategy with a tax level less than $(1-\delta)\,\overline{\tau}$, any tax $\tau < (1-\delta)\,\overline{\tau}$ cannot be supported in SSPE without investment. Also, the following strategies are best response to each other and constitute an SSPE. In every period, the politician does not invest and collects the tax $(1-\delta)\,\overline{\tau}$, and the voter re–elects the politician if and only if he collects a tax less than or equal to $(1-\delta)\,\overline{\tau}$. Thus, there exists an SSPE with the politician's policy $((1-\delta)\,\overline{\tau}, 0)$. $\square$

Lemma 13 *There exists an SSPE with investment if and only if* $\tau^* \leq \overline{\tau}$.

Proof: Lemma 7 shows that if $\tau^* > \overline{\tau}$, then for any strategy of the voter, the politician prefers not to invest. Thus, there does not exist an SSPE with investment. If $\tau^* \leq \overline{\tau}$, then the following strategies are best response to each other and constitute an SSPE. In every period, the politician invests and collects the tax $\overline{\tau}$ and the voter re–elects the politician if and only if the income is greater than or equal to y^*. Thus, there exists an SSPE with investment. $\square$

The equilibria of the no commitment game can be described as follows.

Proposition 7 *When the voter cannot observe the investment policy of the politician, the no commitment game has multiple SSPE. If* $\tau^* \leq \overline{\tau}$ *and* $\phi(\tau^*) \geq y^*$, *then in these equilibria, either the politician invests and collects a tax* $\tau \geq \tau^*$ *or the politician does not invest and collects a tax* $\tau \geq (1-\delta)\overline{\tau}$. *If* $\tau^* \leq \overline{\tau}$ *and* $\phi(\tau^*) < y^*$, *then in these equilibria, either the politician invests and collects a tax* $\tau \geq \tau^{**}$ *or the politician does not invest and collects a tax* $\tau \geq (1-\delta)\overline{\tau}$. *If* $\tau^* > \overline{\tau}$, *then in these equilibria, the politician does not invest and collects a tax* $\tau \geq (1-\delta)\overline{\tau}$.

If the voter is unable to commit, since her strategy to remove the politician from office is not a credible threat for the politician, there exist multiple equilibria. The best equilibria for the voter are those that have the same outcome with the equilibria of the commitment game.

1.5 A Numerical Example

Let $y \in [0,2]$ and conditional probability density functions be $f(y|0) = 1-\frac{y}{2}$ and $f\left(y|\hat{I}\right) = \frac{y}{2}$. Accordingly, the densities have the strict MLRP and the voter's expected benefit from the investment is equal to $\frac{2}{3}$. Thus, if the cost of investment is less than $\frac{2}{3}$, then the investment decision of the politician is socially optimal. Assume that $\overline{\tau} = 0.5$ and $\delta = 0.8$.

Note that $(1-\delta)\overline{\tau}$ is equal to 0.1. Hence, there is no SSPE without investment in which the politician collects a tax less than 0.1 and there is no SSPE with investment in which the politician collects a tax less than $0.1 + \hat{I}$.

If $\hat{I} \in \left(0.4, \frac{2}{3}\right]$, then $\overline{\tau} < \frac{\hat{I}}{\delta}$, and thus the increase in the politician's expected future payoff because of the investment is less than the cost of investment. Consequently, there is no SSPE with investment. Irrespective of the transparency of the political system, in SSPE the politician does not invest, collects the tax 0.1, and is re–elected. We derive the equilibrium for the values of $\hat{I}$ in the interval $[0, 0.4]$.

A. The Commitment Game

A.1. The Case of a Transparent Political System

In SSPE, the politician invests, collects the tax $0.1 + \hat{I}$, and is never replaced. The politician's payoff is equal to 0.5.

A.2. The Case of a Non–Transparent Political System:

The functions ϕ and ψ can be derived as follows.

$$\phi(\tau) = \begin{cases} 0 & \text{if } \tau \in \left[0, 0.1 + \hat{I}\right] \\ 2\sqrt{\frac{\tau - 0.1 - \hat{I}}{0.4}} & \text{if } \tau \in \left[0.1 + \hat{I}, 0.5\right] \end{cases}$$

$$\psi(y) = 0.25\hat{I}\frac{\left(1 + 4y - y^2\right)}{\left(y - \frac{y^2}{2}\right)} \text{ for } y \in (0, 2)$$

It can be demonstrated that $\psi(y)$ is convex on its domain and has its minimum value at $y^* = 0.618$. Thus, τ^* can be written as follows.

$$\tau^* = \psi(y^*) = 1.809\hat{I}$$

Since there is no strategy of the voter to induce the politician to invest and to collect a tax less than τ^*, if $\overline{\tau} < \tau^*$, there is no SSPE with investment.

A.2.1. The case of $\hat{I} \in (0.276, 0.4]$

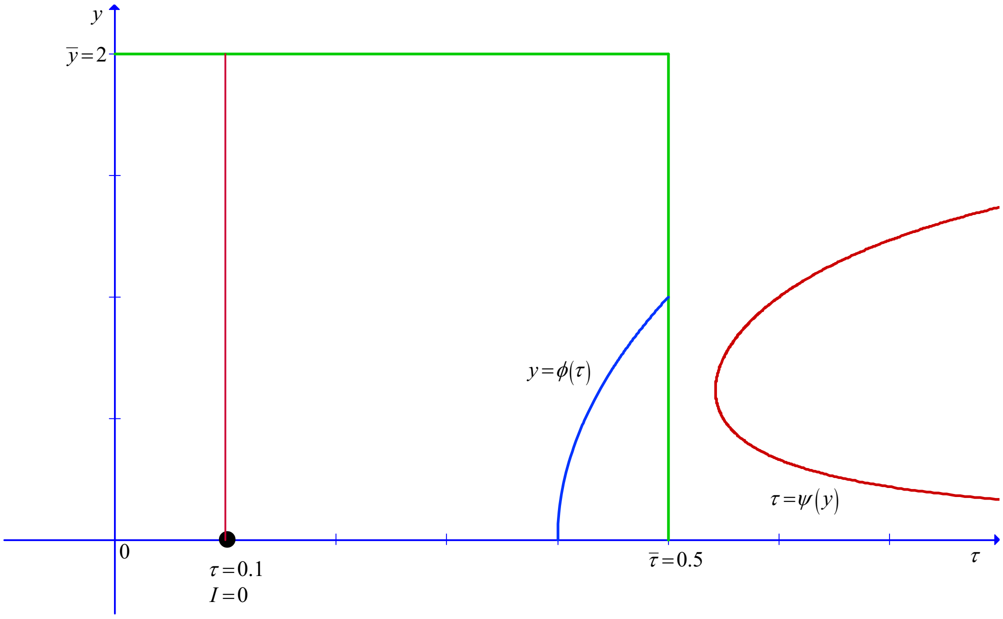

Figure 1: Equilibrium ($\hat{I} = 0.3$)

If $\hat{I} > 0.276$, then $\overline{\tau} < \tau^*$, and consequently, there is no SSPE with investment. In SSPE, the politician does not invest, collects the tax 0.1, and is re–elected. The politician's payoff is equal to 0.5. Figure 1 shows the equilibrium for $\hat{I} = 0.3$.

A.2.2. The case of $\hat{I} \in (0.171, 0.276]$

If the cost of investment is in that interval, then $\phi(\tau^*) \geq y^*$. Thus, the politician prefers the strategy $\left(\tau^*, \hat{I}\right)$ to the strategy $(\overline{\tau}, 0)$. τ^* is the lowest tax that the voter can induce the politician to collect and also to invest. Since the expected benefit of the investment is $\frac{2}{3}$, it is easily seen that the investment is worthwhile. Consequently, in SSPE, the politician invests, collects the tax $\tau^* = 1.809\hat{I}$, and is re–elected if the income is greater than 0.618. The re–election probability of the politician is 0.90 and his expected payoff is equal to $2.927\hat{I}$, which is greater than 0.5.

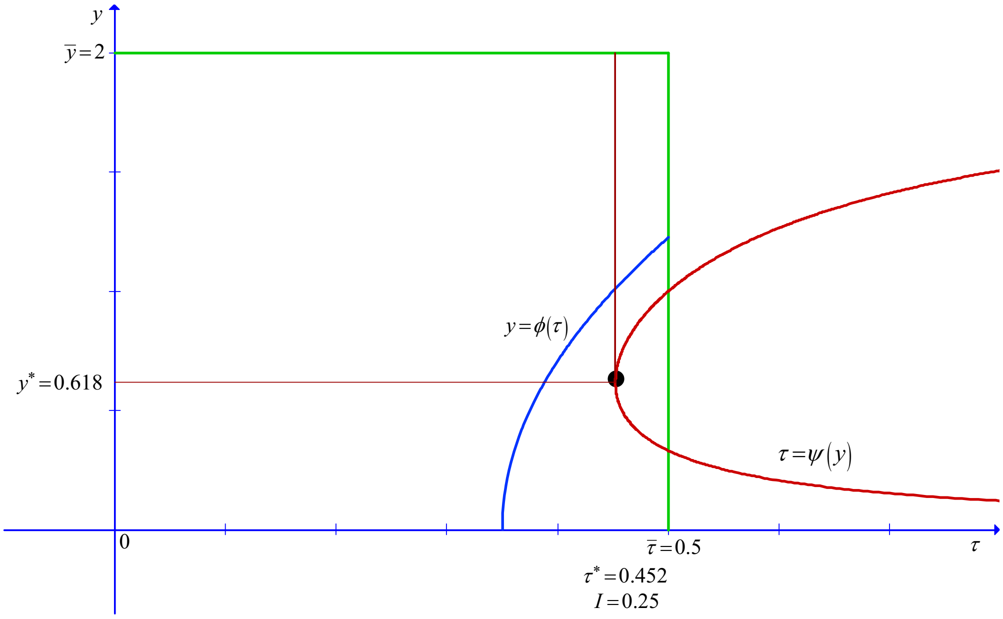

Figure 2: Equilibrium ($\hat{I} = 0.25$)

The equilibrium for $\hat{I} = 0.25$ is presented in Figure 2 where the politician invests, collects the tax $\tau^* = 0.452$, and is re–elected if the income is greater than 0.618. The expected payoff of the politician is equal to 0.732.

A.2.3. The case of $\hat{I} \leq 0.171$

If the cost of investment is less than or equal to 0.171, then $\phi(\tau^*) < y^*$. The politician prefers the strategy $(\overline{\tau}, 0)$ to the strategy $\left(\tau^*, \hat{I}\right)$. τ^{**} is the lowest tax that the voter can induce the politician to collect and also to invest. τ^{**} can be found by using (2). For example, if $\hat{I} = 0.1$, then $\tau^{**} = 0.208$ and the politician is re–elected with probability 0.98. If $\hat{I} = 0.15$, then $\tau^{**} = 0.275$ and the politician is re–elected with probability 0.938.

The investment is worthwhile, and in SSPE, the politician invests, collects the tax τ^{**}, and is re–elected with probability $1 - \frac{\phi^2(\tau^{**})}{4}$. The politician's expected payoff is equal to 0.5.

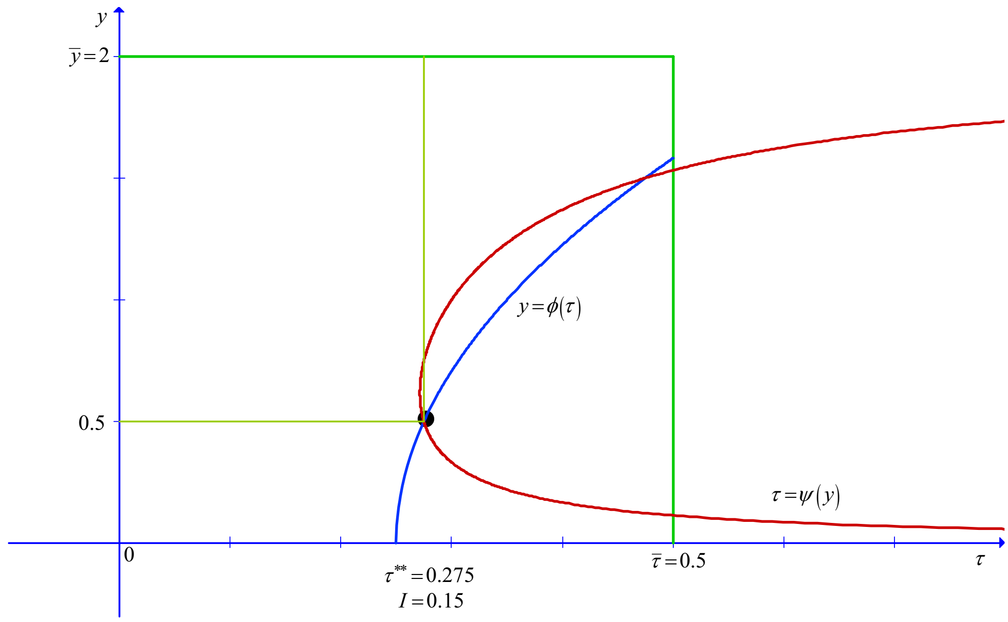

Figure 3: Equilibrium ($\hat{I} = 0.15$)

The equilibrium for $\hat{I} = 0.15$ is shown in Figure 3, where the politician invests, collects the tax $\tau^{**} = 0.275$, and is re–elected if the income is greater than 0.5. The re–election probability of the politician is 0.938 and his expected payoff is equal to 0.5.

B. The No Commitment Game

B.1. The Case of a Transparent Political System:

In SSPE, either the politician does not invest and collects a tax $\tau \in [0.1, 0.5]$ or the politician invests and collects a tax $\tau \in \left[0.1 + \hat{I}, 0.5\right]$.

B.2. The Case of a Non–Transparent Political System:

If $\hat{I} > 0.276$, then $\overline{\tau} < \tau^*$, and consequently, there is no SSPE with investment. In SSPE, the politician does not invest and collects a tax $\tau \geq 0.1$. If $\hat{I} \in (0.171, 0.276]$, then in SSPE, the politician either does not invest and collects a tax $\tau \geq 0.1$ or invests and collects a tax

$\tau \geq \tau^*$. If $\hat{I} \leq 0.171$, then in SSPE, the politician either does not invest and collects a tax $\tau \geq 0.1$ or invests and collects a tax $\tau \geq \tau^{**}$.

1.6 Conclusion

This chapter considered the effects of asymmetric information on the public control of politicians in a world where the politicians' pre–election promises are not credible. We presented a model with identical politicians and a representative voter whose interests conflict with those of the politicians. The politician makes decisions on two policies that affect the voter's welfare. The voter observes only one of the politician's policies and her strategy whether or not to re–elect the politician depends on both her welfare and the politician's decision on the observable policy.

In a transparent political system, the politician makes socially optimal decisions. However, in a non–transparent political system, depending on the power of the politician, the effect of politician's unobserved policy to the welfare of the voter, and the politician's discount rate for future payoffs, either socially optimal decisions are not taken by the politician or if taken, the politician would extract more rent in each period. In the latter case, politicians are also replaced more frequently.

The voter prefers the political system to be more transparent. Also, we demonstrated that the politician may prefer the lower per–period payoff in a transparent political system to the higher per–period payoff in a non–transparent political system, since he is re–elected more frequently in the former one. Consequently, a reform that would make the political system more transparent should be supported by the public and may also be welcomed by the politician.

1.7 Appendix

Proof of Proposition 1

The proof is based on the following lemma.

Lemma 14 *Assume that in an SSPE, the politician's strategy is* (τ', I') *and he is re–elected if the income* $y \in J$*, where the greatest lower bound of the set* J *is* $\underline{y}$*. Then,* $J = \left[\underline{y}, \overline{y}\right]$.

Proof. First, we will show that if $I' = 0$, then $J = \left[\underline{y}, \overline{y}\right]$. Second, we will show that if $I' = \hat{I}$, then $J = \left[\underline{y}, \overline{y}\right]$. Let $J_0 \subset \left[\underline{y}, \overline{y}\right]$ with $J_0 \neq \varnothing$ and $J = \left[\underline{y}, \overline{y}\right] \setminus J_0$.

First, assume that $I' = 0$. In SSPE, the politician does not invest and collects the tax τ'. Since the politician prefers the strategy $(\tau', 0)$ to strategy $(\overline{\tau}, 0)$,

$$\frac{\tau'}{1 - \delta \int_J f(y|0)\, dy} \geq \overline{\tau}.$$

Since $J_0 \neq \varnothing$, $\int_{J_0} f(y|0)\, dy > 0$. So, there exists a tax level $\tau'' < \tau'$ such that

$$\frac{\tau''}{1 - \delta \int_{\underline{y}}^{\overline{y}} f(y|0)\, dy} > \frac{\tau'}{1 - \delta \int_J f(y|0)\, dy}.$$

Therefore, the voter can induce the politician to collect a lower tax by choosing the strategy "re–elect the politician if and only if the collected tax is τ'' and the income $y \in \left[\underline{y}, \overline{y}\right]$." Thus, if $J \neq \left[\underline{y}, \overline{y}\right]$, then the voter's strategy cannot be an equilibrium strategy.

Second, assume that $I' = \hat{I}$. In SSPE, the politician invests and collects the tax τ'. Since the politician prefers the strategy $\left(\tau', \hat{I}\right)$ to both strategy $(\overline{\tau}, 0)$ and strategy $(\tau', 0)$, it can be written as follows.

$$\frac{\tau' - \hat{I}}{1 - \delta \int_J f\left(y|\hat{I}\right) dy} \geq \overline{\tau} \tag{3}$$

$$\frac{\tau' - \hat{I}}{\tau'} \geq \frac{1 - \delta \int_J f\left(y|\hat{I}\right) dy}{1 - \delta \int_J f\left(y|0\right) dy} \tag{4}$$

Since the greatest lower bound of J_0 is greater than $\underline{y}$, there exists a subset of J, say J', and a subset of J_0, say J_0', such that:[1]

$$\sup\left(J'\right) < \inf\left(J_0'\right) \text{ and} \tag{5}$$

$$\int_{J'} f\left(y|0\right) dy = \int_{J_0'} f\left(y|0\right) dy. \tag{6}$$

Strict MLRP requires that

$$\frac{\int_{J'} f\left(y|\hat{I}\right) dy}{\int_{J'} f\left(y|0\right) dy} < \frac{f\left(\sup(J')|\hat{I}\right)}{f\left(\sup(J')|0\right)} \text{ and } \frac{f\left(\inf(J_0')|\hat{I}\right)}{f\left(\inf(J_0')|0\right)} < \frac{\int_{J_0'} f\left(y|\hat{I}\right) dy}{\int_{J_0'} f\left(y|0\right) dy}. \tag{7}$$

Since $\sup\left(J'\right) < \inf\left(J_0'\right)$, strict MLRP implies that

$$\frac{f\left(\sup(J')|\hat{I}\right)}{f\left(\sup(J')|0\right)} < \frac{f\left(\inf(J_0')|\hat{I}\right)}{f\left(\inf(J_0')|0\right)}. \tag{8}$$

Consequently, (7) and (8) implies that

$$\frac{\int_{J'} f\left(y|\hat{I}\right) dy}{\int_{J'} f\left(y|0\right) dy} < \frac{\int_{J_0'} f\left(y|\hat{I}\right) dy}{\int_{J_0'} f\left(y|0\right) dy}. \tag{9}$$

Then, (6) and (9) requires that

$$\int_{J'} f\left(y|\hat{I}\right) dy < \int_{J_0'} f\left(y|\hat{I}\right) dy. \tag{10}$$

[1] $\inf(J)$ denotes the infimum (greatest lower bound) of set J and $\sup(J)$ denotes the supremum (least upper bound) of set J.

Let us define the set J^* as follows.

$$J^* = J \backslash J' \cup J_0'$$

Then, (6) implies that

$$1 - \delta \int_{J^*} f(y|0)\, dy = 1 - \delta \int_{J} f(y|0)\, dy, \tag{11}$$

and (10) implies that

$$1 - \delta \int_{J^*} f\left(y|\hat{I}\right) dy < 1 - \delta \int_{J} f\left(y|\hat{I}\right) dy. \tag{12}$$

From (11) and (12), it can be written as:

$$\frac{1 - \delta \int_{J^*} f\left(y|\hat{I}\right) dy}{1 - \delta \int_{J^*} f(y|0)\, dy} < \frac{1 - \delta \int_{J} f\left(y|\hat{I}\right) dy}{1 - \delta \int_{J} f\left(y|\hat{I}\right) dy}. \tag{13}$$

Therefore, (3),(4), (12) and (13) implies that there exists a tax level $\tau''' < \tau'$ satisfying the following two conditions.

$$\begin{aligned} \frac{\tau''' - \hat{I}}{1 - \delta \int_{J^*} f\left(y|\hat{I}\right) dy} &\geq \overline{\tau} \\ \frac{\tau''' - \hat{I}}{\tau'''} &\geq \frac{1 - \delta \int_{J^*} f\left(y|\hat{I}\right) dy}{1 - \delta \int_{J^*} f(y|0)\, dy} \end{aligned}$$

Therefore, if the voter chooses the strategy "re–elect the politician if and only if the collected tax is τ''' and the income $y \in J^*$," the politician will collect a lower tax than τ' and still invest. Thus, if, $J \neq [y_0, \overline{y}]$, then the voter's strategy cannot be an equilibrium strategy. □

Suppose that in an SSPE, the politician's strategy is (τ', I') and he is re–elected if the income $y \in \left[\underline{y}, \overline{y}\right]$. The same outcome can be generated by an SSPE in which the voter's strategy is "re–elect the politician if and only if the collected tax is τ' and the income $y \in \left[\underline{y}, \overline{y}\right]$." Consequently, any SSPE outcome can be generated by an SSPE with monotone strategies. □

Proof of Lemma 1

Since the players's strategies are stationary, $U\left(\rho|\varkappa\right)$ can be written as

$$U\left(\rho|\varkappa\right) = \tau - I + \delta\alpha\left(\rho|\varkappa\right)U\left(\rho|\varkappa\right). \tag{14}$$

By solving (14) for $U\left(\rho|\varkappa\right)$ and substituting $\alpha\left(\rho|\varkappa\right)$ with $\left(1 - F\left(g(\tau)|I\right)\right)$, we obtain

$$U\left(\rho|\varkappa\right) = \frac{\tau - I}{1 - \delta\left(1 - F\left(g(\tau)|I\right)\right)}.$$

□

Proof of Proposition 2

When the politician invests, his expected payoff is maximized if he collects the maximum allowable tax, $\overline{\tau}$, and is re–elected every period. Therefore, $U\left(\left(\tau, \hat{I}\right)|\varkappa\right)$ has the following upper bound.

$$U\left(\left(\tau, \hat{I}\right)|\varkappa\right) \leq \frac{\overline{\tau} - \hat{I}}{1 - \delta}$$

The politician can get an expected payoff at least $\overline{\tau}$ by choosing ρ_0. Therefore, if

$$\frac{\overline{\tau} - \hat{I}}{1 - \delta} < \overline{\tau}, \tag{15}$$

then the politician does not invest. The inequality in (15) can be written as

$$\overline{\tau} < \frac{\hat{I}}{\delta}.$$

□

Proof of Lemma 2

If the politician does not invest and collects the tax $(1-\delta)\,\overline{\tau}$ every period, then his expected payoff is equal to

$$U\left(\left((1-\delta)\,\overline{\tau},0\right)|\varkappa\right)=\frac{(1-\delta)\,\overline{\tau}}{1-\delta\left(1-F\left(g(\tau)|I\right)\right)}.$$

Since the re–election probability of the politician is less than or equal to 1, i.e., $[1-F\left(g(\tau)|I\right)]\leq 1,$

$$\frac{(1-\delta)\,\overline{\tau}}{1-\delta\left(1-F\left(g(\tau)|I\right)\right)}\leq\overline{\tau}. \tag{16}$$

If the politician collects a tax lower than $(1-\delta)\,\overline{\tau}$, then the inequality in (16) is strict. Therefore, for any strategy of the voter, the politician prefers to choose ρ_0 to the strategy $(\tau,0)$ for $\forall\tau<(1-\delta)\,\overline{\tau}$. Consequently, there does not exist an SSPE without investment in which the tax is less than $(1-\delta)\,\overline{\tau}$. □

Proof of Lemma 3

If the politician invests and collects the tax $\hat{I}+(1-\delta)\,\overline{\tau}$ every period, then his expected payoff is equal to

$$U\left(\left(\hat{I}+(1-\delta)\,\overline{\tau},\hat{I}\right)|\varkappa\right)=\frac{(1-\delta)\,\overline{\tau}}{1-\delta\left(1-F\left(g(\tau)|I\right)\right)}\leq\overline{\tau}. \tag{17}$$

If the politician collects a lower tax than $\hat{I}+(1-\delta)\,\overline{\tau}$, then the inequality in (17) is strict. Therefore, for any strategy of the voter, the politician prefers to choose ρ_0 to the strategy $(\tau,0)$ for $\forall\tau<\hat{I}+(1-\delta)\,\overline{\tau}$. Consequently, there does not exist an SSPE with investment in which the tax is less than $\hat{I}+(1-\delta)\,\overline{\tau}$. □

Proof of Proposition 3

The cost of inducing the politician to invest is equal to the cost of investment. Since the cost of investment is lower than the expected benefit of investment, the voter induces the politician to invest. The voter can induce the politician to collect the tax $\hat{I} + (1-\delta)\overline{\tau}$ and also to invest by re–electing him with probability 1 if and only if he chooses this policy in each period. Consequently, in SSPE, the politician invests, collects the tax $\hat{I} + (1-\delta)\overline{\tau}$, and is re–elected every period. $\square$

Proof of Lemma 4

If the politician collects a tax less than $(1-\delta)\overline{\tau}$, then for any strategy of the voter, his expected payoff is less than $\overline{\tau}$. Therefore, for any strategy of the voter, the politician prefers to choose ρ_0 to the strategy $(\tau, 0)$ for $\forall\tau < (1-\delta)\overline{\tau}$. Consequently, there does not exist an SSPE without investment where the tax is less than $(1-\delta)\overline{\tau}$. $\square$

Proof of Lemma 5

Assume that $\tau \in \left[\hat{I} + (1-\delta)\overline{\tau}, \overline{\tau}\right]$. Given the voter's strategy $\varkappa$, the politician's expected payoff when he chooses ρ_0 and the strategy $\left(\tau, \hat{I}\right)$ are as follows.

$$
\begin{aligned}
U(\rho_0|\varkappa) &= \frac{\overline{\tau}}{1-\delta\left(1-F\left(g(\overline{\tau})|0\right)\right)} \geq \overline{\tau} \\
U\left(\left(\tau, \hat{I}\right)|\varkappa\right) &= \frac{\tau - \hat{I}}{1-\delta\left(1-F\left(g(\tau)|\hat{I}\right)\right)}
\end{aligned}
$$

First, note that if $g(\tau) = \phi(\tau)$, then $U\left(\left(\tau, \hat{I}\right)|\varkappa\right) = \overline{\tau}$. Second, $U\left(\left(\tau, \hat{I}\right)|\varkappa\right)$ is strictly decreasing in $g(\tau)$. Consequently,

$$
g(\tau) > \phi(\tau) \Rightarrow U\left(\left(\tau, \hat{I}\right)|\varkappa\right) < \overline{\tau} \leq U\left((\overline{\tau}, 0)|\varkappa\right).
$$

Therefore, if $g(\tau) > \phi(\tau)$, then the politician prefers to choose ρ_0 to the strategy $\left(\tau, \hat{I}\right)$.

$\square$

Proof of Lemma 6

Let us define $\chi : (0, \overline{y}) \rightarrow \mathbb{R}$ as follows.

$$\chi(y) = \frac{1 - \delta + \delta F\left(y | \hat{I}\right)}{1 - \delta + \delta F(y|0)}$$

Note that $\psi(y) = \frac{\hat{I}}{1-\chi(y)}$. Hence, if we prove that $\chi(y)$ has a unique minimum point, it implies that $\psi(y)$ has a unique minimum point.

Note that $\chi'(0) < 0$ and $\chi'(\overline{y}) > 0$. Hence $\chi(y)$ has at least a minimum point. Fix a point y^* in which $\chi(y)$ is non–decreasing. Let us say $\chi(y^*) = \pi$. Since $\chi(y)$ is non–decreasing at y^*, for an $\varepsilon \rightarrow 0^+$,

$$\chi(y^* + \varepsilon) \geq \chi(y^*).$$

Hence, it can be written as

$$\chi(y^* + \varepsilon) = \frac{1 - \delta + \delta F\left(y^* + \varepsilon | \hat{I}\right)}{1 - \delta + \delta F(y^* + \varepsilon|0)} \geq \pi. \tag{18}$$

By using Taylor approximation of $F(y|\cdot)$ of order one at y^*, inequality in (18) can be rewritten as follows.

$$\frac{1 - \delta + \delta F\left(y^* | \hat{I}\right) + \delta\varepsilon f\left(y | \hat{I}\right)}{1 - \delta + \delta F(y^*|0) + \delta\varepsilon f(y|0)} \geq \pi \tag{19}$$

Since $\chi(y^*) = \pi$, by substituting $\left[1 - \delta + \delta F\left(y^* | \hat{I}\right)\right]$ with $\pi\left[1 - \delta + \delta F(y^*|0)\right]$ in (19) and arranging the terms, we obtain

$$\frac{f\left(y^* | \hat{I}\right)}{f(y^*|0)} \geq \pi. \tag{20}$$

(20) and strict MLRP imply that for $\forall t \in (0, \overline{y} - y^*)$,

$$f\left(y^* + t|\hat{I}\right) > \pi f\left(y^* + t|0\right). \tag{21}$$

By integrating both sides of (21) over $[y^*, y^* + t]$, we obtain

$$\left[F\left(y^* + t|\hat{I}\right) - F\left(y^*|\hat{I}\right)\right] > \pi\left[F\left(y^* + t|0\right) - F\left(y^*|0\right)\right]. \tag{22}$$

Let us define $\lambda(y^*, t)$ as the difference between LHS and RHS of (22) as follows.

$$\lambda(y^*, t) = \left[F\left(y^* + t|\hat{I}\right) - F\left(y^*|\hat{I}\right)\right] - \pi\left[F\left(y^* + t|0\right) - F\left(y^*|0\right)\right]$$

Note that (22) implies that $\lambda(y^*, t) > 0$.

$\chi(y^* + t)$ can be written as follows.

$$\chi(y^* + t) = \frac{1 - \delta + \delta F\left(y^*|\hat{I}\right) + \delta\left[F\left(y^* + t|\hat{I}\right) - F\left(y^*|\hat{I}\right)\right]}{1 - \delta + \delta F\left(y^*|0\right) + \delta\left[F\left(y^* + t|0\right) - F\left(y^*|0\right)\right]} \tag{23}$$

By substituting

$$\left[1 - \delta + \delta F\left(y^*|\hat{I}\right)\right] \text{ with } \pi\left[1 - \delta + \delta F\left(y^*|0\right)\right], \text{ and}$$

$$\left[F\left(y^* + t|\hat{I}\right) - F\left(y^*|\hat{I}\right)\right] \text{ with } \lambda(y^*, t) + \pi\left[F\left(y^* + t|0\right) - F\left(y^*|0\right)\right]$$

in (23) and by arranging the terms, we obtain

$$\chi(y^* + t) = \chi(y^*) + \lambda(y^*, t)\left[\frac{\delta}{1 - \delta + \delta F\left(y^* + t|0\right)}\right]. \tag{24}$$

Since $\lambda(y^*, t) > 0$, (24) implies that $\chi(y^* + t) > \chi(y^*)$ for $\forall t \in (0, \overline{y} - y^*)$. So if $\chi(y)$ is non–decreasing at point y^*, then it is strictly increasing when $y > y^*$. Also, since $\chi'(0) < 0$

and $\chi'(\overline{y}) > 0$, there exists a point in the interval $(0, \overline{y})$ where $\chi(y)$ is non–decreasing. Consequently, $\chi(y)$ has a unique minimum point. $\square$

Proof of Lemma 7

For any strategy of the voter, if the politician prefers the strategy $(\tau, 0)$ to the strategy $\left(\tau, \hat{I}\right)$ for $\forall \tau \in [0, \overline{\tau}]$, then the voter is not able to induce the politician to invest, and thus there is no SSPE with investment. Therefore, a necessary condition for the existence of an SSPE with investment is that for at least a tax $\tau' \in [0, \overline{\tau}]$ and a strategy $\varkappa$ of the voter

$$U\left(\left(\tau', \hat{I}\right) | \varkappa\right) \geq U\left((\tau', 0) | \varkappa\right). \tag{25}$$

(25) can be written as follows.

$$\frac{\tau' - \hat{I}}{1 - \delta\left(1 - F\left(g(\tau')|\hat{I}\right)\right)} \geq \frac{\tau'}{1 - \delta\left(1 - F\left(g(\tau')|0\right)\right)} \tag{26}$$

By arranging the terms in (26), we obtain that it is equivalent to

$$\tau' \geq \psi\left(g(\tau')\right). \tag{27}$$

Lemma 6 shows that there is a unique minimum of $\psi(\cdot)$ and τ^* denotes the minimum value of $\psi(\cdot)$. Therefore, if $\overline{\tau} < \tau^*$, then the maximum possible value of the LHS of (27) is less than the minimum value of the RHS of (27). Consequently, the necessary condition for the existence of an SSPE with investment does not hold. Hence, if $\overline{\tau} < \tau^*$, then there does not exist an SSPE with investment. $\square$

Proof of Lemma 8

By construction of $\psi(\cdot)$, the lowest tax τ that the politician prefers the strategy $\left(\tau, \hat{I}\right)$ to the strategy $(\tau, 0)$ is τ^*. Given the voter's strategy $\varkappa^*$, "re–elect the politician if and only if the collected tax is τ^* and the income $y \geq y^*$," the politician prefers the strategy $\left(\tau^*, \hat{I}\right)$ to the strategy $(\tau^*, 0)$. Thus, to prove the lemma we have to show that if $\phi(\tau^*) \geq y^*$, then the politician also prefers the strategy $\left(\tau^*, \hat{I}\right)$ to ρ_0 under the voter's strategy $\varkappa^*$.

Given the voter's strategy $\varkappa^*$, the politician's payoff when he chooses ρ_0 is equal to $\overline{\tau}$. The politician's payoff when he chooses the strategy $\left(\tau^*, \hat{I}\right)$ is as follows.

$$U\left(\left(\tau^*, \hat{I}\right)|\varkappa^*\right) = \frac{\tau^* - \hat{I}}{1 - \delta\left(1 - F\left(y^*|\hat{I}\right)\right)}$$

If $\phi(\tau^*) \geq y^*$, then

$$U\left(\left(\tau^*, \hat{I}\right)|\varkappa^*\right) \geq \frac{\tau^* - \hat{I}}{1 - \delta\left(1 - F\left(\phi(\tau^*)|\hat{I}\right)\right)} = \overline{\tau}.$$

Consequently, the politician prefers the strategy $\left(\tau^*, \hat{I}\right)$ not only to the strategy $(\tau^*, 0)$, but also to ρ_0. $\square$

Proof of Lemma 9

The proof here is by contradiction. Assume that tax $\tau' < \tau^{**}$ and the voter is able to induce the politician to collect the tax τ' and also to invest. Then, there exists a strategy of the voter such that $g(\tau')$ satisfies the following two conditions.

$$g(\tau') \leq \phi(\tau') \quad (28)$$

$$\tau' \geq \psi(g(\tau')) \quad (29)$$

The condition in (28) is necessary for the politician to prefer the strategy $\left(\tau', \hat{I}\right)$ to ρ_0. The condition in (29) is necessary for the politician to prefer the strategy $\left(\tau', \hat{I}\right)$ to the strategy $(\tau', 0)$. If one of these two conditions is violated, then the politician will not invest.

$\phi(\cdot)$ is an increasing function, if $\tau' < \tau^{**}$, then $\phi(\tau') < \phi(\tau^{**})$. Therefore, if $g(\tau')$ satisfies the condition in (28), then

$$g(\tau') < \phi(\tau^{**}). \tag{30}$$

Note first that $\psi(y)$ is decreasing when $y < y^*$. Second, $\phi(\tau^{**}) < y^*$ because of the requirement in (2). Consequently, if $g(\tau')$ satisfies the condition in (28), then (30) implies that

$$\psi(g(\tau')) > \psi(\phi(\tau^{**})). \tag{31}$$

Also, $\psi(\phi(\tau^{**})) = \tau^{**}$ because of the requirement in (2). Thus, both (2) and (31) imply that

$$\psi(g(\tau')) > \tau^{**}.$$

Therefore, to satisfy the condition in (29), τ' must be grater than τ^{**}. This is a contradiction to our initial assumption, i.e., $\tau' < \tau^{**}$.

Proof of Proposition 4

Proof of the proposition follows the Lemmas 4, 8 and 9. The reasoning for the replacement of the politician with positive probability while investing is the following. In SSPE, if the politician invests and collects the tax τ^*, then he is removed from office if the income is less than y^*. Thus, he is replaced with probability $F(y^*|I) > 0$. In SSPE, if the politician invests and collects the tax τ^{**}, then he is removed from office if the income is less than $\phi(\tau^{**})$. Thus, the politician is replaced with probability $F(\phi(\tau^{**})|I) > 0$.

Proof of Proposition 5

In a transparent political system, the politician invests, collects the tax $\hat{I} + (1-\delta)\overline{\tau}$, and is re–elected. His per–period payoff is $(1-\delta)\overline{\tau}$ and his expected payoff is $\overline{\tau}$. The voter's expected payoff is $\int yf\left(y|\hat{I}\right)dy - \left[\hat{I} + (1-\delta)\overline{\tau}\right]$. In a non–transparent political system, in SSPE, the following occurs.

i. If the politician does not invest, then he collects the tax $(1-\delta)\overline{\tau}$ and is re–elected. His per–period payoff is $(1-\delta)\overline{\tau}$ and his expected payoff is $\overline{\tau}$. Therefore, the politician's per–period and expected payoffs are equal to those in the case of a transparent political system. The voter loses the expected benefit of the investment, but pays less tax when compared to a transparent political system in each period. Therefore, the expected welfare loss of the voter is $\int yf\left(y|\hat{I}\right)dy - \int ydf\left(y|0\right)dy - \hat{I}$ in each period.

ii. If the politician invests in SSPE and $\phi\left(\tau^*\right) < y^*$, then he collects the tax τ^{**} and is re–elected with probability $\alpha\left(\left(\tau^{**},\hat{I}\right)|\varkappa\right)$, where

$$\begin{aligned}\alpha\left(\left(\tau^{**},\hat{I}\right)|\varkappa\right) &= 1 - F\left(\phi\left(\tau^{**}\right)|I\right) \\ &= \frac{\overline{\tau} + \hat{I} - \tau^{**}}{\delta\overline{\tau}}.\end{aligned}$$

By plugging $\alpha\left(\left(\tau^{**},\hat{I}\right)|\varkappa\right)$ into the expected payoff function in lemma 1, the politician's expected payoff can be found as $\overline{\tau}$, which is the same with the politician's payoff in a transparent political system. His per–period payoff is $\tau^{**} - \left[\hat{I} + (1-\delta)\overline{\tau}\right]$ which is more than that in the transparent political system. Therefore, asymmetric information causes a transfer of $\tau^{**} - \left[\hat{I} + (1-\delta)\overline{\tau}\right]$ from the voter to the politician in each period.

iii. If the politician invests in SSPE and $\phi\left(\tau^*\right) \geq y^*$, then the politician collects the tax τ^*

and is re–elected with probability $\alpha\left(\left(\tau^*,\hat{I}\right)|\varkappa\right)$, where

$$\begin{aligned}\alpha\left(\left(\tau^*,\hat{I}\right)|\varkappa\right) &= 1-F\left(y^*|I\right) \\ &\geq \left[1-F\left(\phi\left(\tau^*\right)|I\right)\right]=\frac{\overline{\tau}+\hat{I}-\tau^*}{\delta\overline{\tau}}.\end{aligned}$$

Both the per–period and the expected payoff of the politician is greater when compared to a transparent political system. Asymmetric information causes a transfer of $\tau-\left[\hat{I}+(1-\delta)\,\overline{\tau}\right]$ from the voter to the politician in each period. $\square$

Proof of Proposition 7

Lemma 10 reveals that there does not exist an SSPE without investment in which the politician collects a tax less than $(1-\delta)\,\overline{\tau}$. Also, any tax $\tau\geq(1-\delta)\,\overline{\tau}$ can be supported in an SSPE without investment with the following strategies of the players: In every period, the politician does not invest and collects the tax τ and the voter re–elects the politician if and only if the collected tax is less than or equal to τ.

To derive the SSPE with investment, we employ the results of Lemmas 7, 8 and 9. Lemma 7 shows that if $\tau^*>\overline{\tau}$, then for any strategy of the voter, the politician's best response is not to invest. Consequently, there is no SSPE with investment.

Lemma 8 indicates that if $\tau^*\leq\overline{\tau}$ and $\phi\left(\tau^*\right)\geq y^*$, then for any strategy of the voter, the politician does not invest while collecting a tax less than τ^*. Any tax $\tau\geq\tau^*$ can be supported in an SSPE with investment by the following strategies of the players: The politician invests and collects the tax τ in every period. The voter re–elects the politician if and only if the collected tax is less than or equal to τ and the income $y\geq y^*$ in each period.

Lemma 9 shows that if $\tau^* \leq \overline{\tau}$ and $\phi(\tau^*) < y^*$, then for any strategy of the voter, the politician does not invest while collecting a tax less than τ^{**}. Any tax $\tau \geq \tau^{**}$ can be supported in an SSPE with investment by the following strategies of the players: The politician invests and collects the tax τ in every period. The voter re–elects the politician if and only if the collected tax is less than or equal to τ and the income $y \geq \phi(\tau^{**})$ in each period. $\square$

Chapter 2: Political Accountability and Provincial Size

2.1 Introduction

After a politician is elected, he has the authority to choose his policies for a certain time interval. In doing so, he may ignore his campaign promises and instead choose policies that serve his own interests. At the same time, the electorate may not observe all the policies instituted by the politician. As a consequence of the power and information superiority of the politician, political moral hazard arises.

In such a setting, the next election is the only device for electorate to control the politician. If the electorate employs a retrospective voting rule, the threat of losing office can provide incentives to the politician to act in the interests of the electorate regardless of any personal objectives. If the politician is accountable to the electorate, then the electorate will re–elect him and allow him to stay in office. If the politician is not accountable to the electorate, then the electorate will replace him. Therefore, as political accountability increases, we expect politicians to be replaced less frequently.

The purpose of this chapter is to investigate the relation between political accountability and size of the electorate by using the re–election probabilities of politicians.

The central idea of this chapter can be stated as follows. The electorate re–elects the politician and allows him to stay in office if and only if the politician is accountable. In more highly populated provinces, the signal of the electorate about the policies of the politician is noisier. Consequently, in these provinces, the politicians are less able to justify their policies, and so they are less accountable to the electorate and are voted out of offices more frequently. This idea leads to an obvious empirical implication, i.e., the re–election probability of a politician decreases in size of the province.

First, a model in which the probability of being re–elected is decreasing in size of the province is presented. Then, we test this prediction of the model. In doing so, we use a panel of nine municipal elections for 81 provinces of Turkey for the period 1963–2004. After controlling province specific effects, we observe that the re–election probability of the politician decreases in size of the province. One percent increase in population of a province leads to a reduction in the re–election probability of the politician by 0.31 percent.

In our model, there is an elected politician in each province who can use the budget of the province to increase social welfare or to enrich himself. The politician spends a portion of the budget to increase social welfare. Budgets of the provinces are proportional to their population size: the greater the population size, the larger is the budget. The welfare of the electorate stochastically depends on the rate of budget that is spent for increasing social welfare. If the politician spends more to increase social welfare, it is more likely to have higher welfare. If the politicians who control the provinces spend at the same rate, then the amount that is spent to increase social welfare is greater in more highly populated provinces. By considering the fact that it is more difficult to manage a higher level of spending, we assume that the uncertainty in welfare is grater in the more highly populated provinces.

The electorate observes the welfare, but cannot observe how the politician uses the budget. Therefore, the level of welfare is a signal for the electorate about the policy of the politician. In more highly populated provinces, the greater uncertainty in welfare implies a noisier signal for the electorate. We assume that the electorate can coordinate a decision rule as to whether to re–elect the politician. A representative voter adopts a simple retrospective voting rule: re–elect the politician if and only if the welfare is more than a cut–off level.

We analyzed two variants of the model. First, we assume that the voter is unable to commit to a strategy. In this case, any spending rate for increasing welfare up to a threshold value can be sustained in equilibrium. Second, we assume that the voter is able to commit to a strategy. In such case, there exists a unique equilibrium. Given the equilibrium cut–off level of the voter and the policy of the politician, depending on the realization of welfare, the politician is re–elected with some probability. By analyzing the effect of the size of the province to the equilibrium re–election probability of the politician, we find that the re–election probability of the politician decreases in size of the province that he controls.

This chapter represents the first investigation of the effect of province (electorate) size on the re–election probabilities of the politicians and political accountability.[2]

The relation of our model to some of the previous models for public control of the politicians is as follows. In the model in first chapter, the politician has a two–dimensional policy in which the policy that is unobserved by the electorate is a discrete variable. In the present chapter, the politician has a one dimensional policy that is unobserved by the electorate and it is a continuous variable. In studies by Ferejohn (1986), Persson Roland and Tabellini (1997), and Person and Tabellini (2000), the politician chooses a policy that affects the utility of the electorate together with a random variable. He observes the random variable before choosing his policy, but the electorate cannot observe it and adopts a retrospective cut–off voting rule. Consequently, in equilibrium, if the realized value of the random

[2] See Alesina and Spolare (2003) for a general discussion on the size of nations.

variable is in some interval, the politician sets his policy to satisfy the required cut–off utility. Otherwise, the politician behaves opportunistically and is voted out of the office. In other words, he certainly knows the result of the next election immediately after he sets his policy. However, in our model, the politician does not observe the random variable – his information superiority to the electorate is restricted only to the knowledge about his policy. Consequently, in equilibrium, the politician is not sure about the result of the next election until the uncertainty is resolved. By modeling this way, we assert that the gain from approaching to the real case is more than the notational cost of it.

The outline of the chapter is as follows. Section 2.2 describes the model. In section 2.3, we derive the equilibrium and describe the predictions of the model. Section 2.4 presents the empirical analysis. Finally, section 2.5 concludes. All the proofs are relegated to the Appendix.

2.2 Model

In every province, there is an elected politician (mayor) who has the power to use the budget of the province. The budget of a province is formed by the payment from the central government and taxes received from the residents of the province. The payments of the central government are proportional to the populations of the provinces and the tax rates are determined by the central government.

Let τ represent the sum of the payment received from the central government for one person and the tax collected from a resident. The budget of a province with population size N is simply $N\tau$. The higher is the population size, the larger is the budget. The politician can utilize the budget to increase social welfare or to enrich himself. He spends some portion of the budget to increase social welfare and retains the rest for his own use. Let $\lambda \in [0, 1]$ be the rate of the budget that is spent for increasing social welfare. The politician's policy is to choose λ and his payoff for the current period is $N\tau (1 - \lambda)$. We assume that the politician and his opponents are identical in terms of ability and preferences. Also, there exists at least one opponent for the politician in every election.

We assume that the electorate can coordinate on the decision whether to re–elect the politician. Thus, a representative voter decides whether to re–elect the politician. Let $\theta \in \mathbb{R}$ represent the welfare of the voter. The welfare of the voter stochastically depends on the politician's policy as follows.

$$\theta = \lambda + \varepsilon$$

where ε is a random variable. If the politician spends more to increase the social welfare, then having higher welfare is more likely. If the politicians who control the provinces choose the same policy, then the amount that is spent to increase social welfare is greater in more highly populated provinces. By considering that it is more difficult to manage higher level of spending, we assume that the uncertainty in welfare is greater in more highly populated provinces. Therefore, the distribution of ε is assumed to be as follows.

$$\varepsilon \sim Normal\left(0, N\sigma^2\right)$$

ε is normally distributed with mean zero and variance $N\sigma^2$, where σ is constant. Let $f_N(\varepsilon)$ and $F_N(\varepsilon)$ denote the probability density function and cumulative distribution function of the normal distribution, with mean zero and variance $N\sigma^2$, respectively.

The timing and information structure of the events are as follows.

1. The politician selects the rate of the budget to spend for increasing social welfare. The voter is unable to observe this policy.

2. The welfare of the voter is realized.

3. Election is held and the voter re–elects the politician or his opponent.

The welfare of the voter acts as a signal about the politician's policy. Since the uncertainty in welfare is greater in more highly populated provinces, the signal of the voter about the politician's policy is noisier there.

Lacking the ability to observe the politician's policy, the voter's decision rule whether or not to re–elect the politician depends on her welfare. At the end of the period, if the welfare of the voter is high enough, then she re–elects the politician. Otherwise, she removes the politician from the office and elects his opponent. Let $\underline{\theta}$ be the threshold welfare in the voter's decision rule that is required for the re–election of the politician. The voter's decision rule can be represented as "re–elect the politician if and only if $\theta \geq \underline{\theta}$."

An important factor to note here is the voter's ability to commit to a decision rule. If the voter is able to commit to a decision rule, then the politician may behave less opportunistically under the voter's "credible threat" of removing him from the office. In the next section, we will analyze both the case of the voter being able to commit to a decision rule and the case of the voter being unable to commit to a decision rule.

Given the decision rule of the voter, the politician has a re–election probability that depends on his policy. Let $\alpha_N \left(\lambda | \underline{\theta}\right)$ be the re–election probability of the politician who controls a province with population size N when he choose policy λ.

$$\alpha_N(\lambda|\underline{\theta}) = 1 - F_N(\underline{\theta} - \lambda) \tag{32}$$

$\alpha_N(\lambda|\underline{\theta})$ is equal to the probability of having the welfare more than the threshold value. First, note that given the decision rule of voter, the re–election probability of the politician is increasing with his policy. Second, the re–election probability of the politician also depends on the population size of the province since it affects the uncertainty in the welfare.

For simplicity, we assume that politicians have an expected value of staying in office and that it is proportional to the budget of the province. For a politician who controls a province with population size N, the expected value of staying in office is equal to $N\tau v$, where v is a constant parameter. The higher the v, the more valuable it is to stay in office. The expected value of leaving office is normalized to zero. This assumption implies that the politician's policy and the voter's decision rule determine the politician's current period payoff and re–election probability, but the expected value of staying in or leaving office is independent of them.

Let δ represent the common discount factor of the politicians and $U_N(\lambda|\underline{\theta})$ denote the expected payoff of a politician who controls a province with population size N if he chooses policy λ and the threshold welfare in the voter's decision rule is $\underline{\theta}$. $U_N(\lambda|\underline{\theta})$ can be written as follows.

$$U_N(\lambda|\underline{\theta}) = N\tau\left[(1-\lambda) + \delta v\left(1 - F_N(\underline{\theta} - \lambda)\right)\right] \tag{33}$$

The expected payoff of the politician consists of two components: The current period's payoff and the discounted value of the expected payoff from the future. The politician's policy determines both the current period's payoff and his re–election probability. As the policy of the politician increases, the current period's payoff decreases, but the expected payoff from the future increases due to the increase in his re–election probability.

Our equilibrium concept is the subgame perfect equilibrium (SPE) in pure strategies. The assumption of having an exogenous value of staying in office implies that the equilibrium strategies of the players are history–independent. Thus, we restrict our attention to the history–independent strategies of the players. The politician's strategy is to choose policy $\lambda \in [0, 1]$ and his associated payoff is $U_N(\lambda|\underline{\theta})$. The voter's strategy is to choose a threshold welfare $\underline{\theta} \in \mathbb{R}$ and to re–elect the politician if and only if $\theta > \underline{\theta}$. The voter's associated payoff is her welfare θ.

Political Accountability

If the politician is accountable to the electorate, then the electorate will re–elect him and allow him to stay in office. On the other hand, if the politician is not accountable to the electorate, then the electorate will replace him. As political accountability increases, politicians will act more in favor of the electorate's interests and less for their private benefits. Thus, politicians' re–election probabilities increase with accountability. As political accountability increases, we expect politicians to have greater re–election probabilities and less turnover in the political system. With this assumption, we compare the political accountability among provinces as follows. Given two provinces, Province A and Province B, if the politicians are replaced less frequently in Province A compared to Province B, then Province A is politically more accountable than Province B. That is to say, if the re–election probability of the politician who controls Province A is greater than his Province

B counterpart, then Province A is politically more accountable than Province B.

2.3 Equilibrium

The ability of the voter to commit to a strategy whether or not to re–elect the politician is crucial in this game. The game in which the voter is able to commit to a strategy is called the "commitment game," while the game in which the voter is unable to commit to a strategy is called the "no commitment game." We analyze the commitment game, followed by the no commitment game.

The Commitment Game

The voter is able to commit to a strategy: She chooses a strategy at the beginning of the period and votes accordingly at the end of the period. Since she is able to commit, losing the office in the case of welfare realizes less than the threshold becomes a credible threat for the politician. Thus, the politician selects his policy by considering the voter's strategy to be credible.

Proposition 8 *There exist two bounds, $\underline{v} < \overline{v}$. If $v \leq \underline{v}$, then in any SPE, the politician's policy is $\lambda = 0$, and if $v \geq \overline{v}$, then in any SPE, the politician's policy is $\lambda = 1$.*

If $v \leq \underline{v}$, then the expected value of staying in office is too low in order to offer any incentive to the politician to spend some portion of the budget for increasing social welfare. For any strategy of the voter and for any $\lambda \in [0, 1]$, the positive effect of a marginal increase in λ to the expected future payoff of the politician due to increase in his re–election probability, is less than the negative effect of it to his current period payoff. Thus, the politician behaves opportunistically for any decision rule of the voter and chooses $\lambda = 0$ in any SPE.

If $v \geq \underline{v}$, then the expected value of staying in office is high enough so that the voter can induce the politician to spend all the budget on increasing social welfare. In any SPE, the politician chooses $\lambda = 1$ to maximize his re–election probability. The values of the bounds $\underline{v}$ and $\underline{v}$ are relegated to the proof of the proposition in the Appendix.

In the rest of this chapter, to focus on the equilibria with politician's interior policy, we restrict our attention to the values of parameter v in the interval $(\underline{v}, \overline{v})$.

To simplify the notation, we define the function $g_N(\cdot)$ as follows.

$$g_N(\varepsilon) = \begin{cases} f_N(\varepsilon) & \text{if } \varepsilon \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$g_N(\varepsilon)$ is simply equal to the probability density function of ε in a province with population size N if ε is non–negative, and equals zero if ε is negative. Note that $g_N(\cdot)$ is decreasing if $\varepsilon \geq 0$, and invertible. Also, let h_N denote the following point on the inverse of $g_N(\cdot)$.

$$h_N = g_N^{-1}\left(\frac{1}{\delta v}\right) \tag{34}$$

The equilibrium of the game is described as follows.

Proposition 9 *If the voter is able to commit, then there exists a unique SPE. In this equilibrium, the politician chooses $\lambda = \theta^* + h_N$, the voter re–elects the politician if and only if $\theta \geq \theta^*$, and consequently, the politician is re–elected with probability $F_N(h_N)$.*

The equilibrium threshold value for welfare, θ^*, is in the interval $(h_N, 1 - h_N)$.[3] The politician's equilibrium policy is greater than θ^* and it is in the interval $(0, 1)$. The equilibrium re–election probability of the politician is $F_N(h_N)$.

[3] The characterization of θ^* is omitted to the proof of the proposition 9 in the Appendix.

The No Commitment Game

Proposition 10 *If the voter is unable to commit, then any policy of the politician, such that* $\lambda \leq \theta^* + h_N$, *can be supported in a SPE.*

For any $\underline{\theta} \in [-h_N, \theta^*]$, if the voter chooses the threshold $\underline{\theta}$, then the the politician's best response is to choose the policy $\lambda = \underline{\theta} + h_N$. If the politician selects the policy $\lambda = \underline{\theta} + h_N$, then a (weakly) optimal strategy of the voter is to set the threshold $\underline{\theta}$. Hence, any policy of the politician, such that $\lambda \leq \theta^* + h_N$, can be supported in an equilibrium.

Note that since the politician and his opponents are identical in terms of their ability and preferences, given the politician's strategy, any strategy of the voter is weakly optimal. Therefore, if the voter is unable to commit, removing the politician from office if the realized welfare is lower than a threshold does not present a credible threat to the politician. Consequently, there exist equilibria where the politician chooses lower policies than the equilibrium policy of the commitment game. The highest policy of the politician that can be sustained in equilibrium is identical to the equilibrium policy of the commitment game.

Henceforth, we assume that the voter can commit to a strategy. Consequently, as proposition 9 implies, there exists a unique equilibrium in which the politician's policy is $\lambda = \theta^* + h_N$ and he is re–elected with probability $F_N(h_N)$. We are now able to compare the re–election probabilities of the politicians among the provinces and comment on the political accountability among provinces.

Proposition 11 *The politicians are replaced more frequently in more highly populated provinces.*

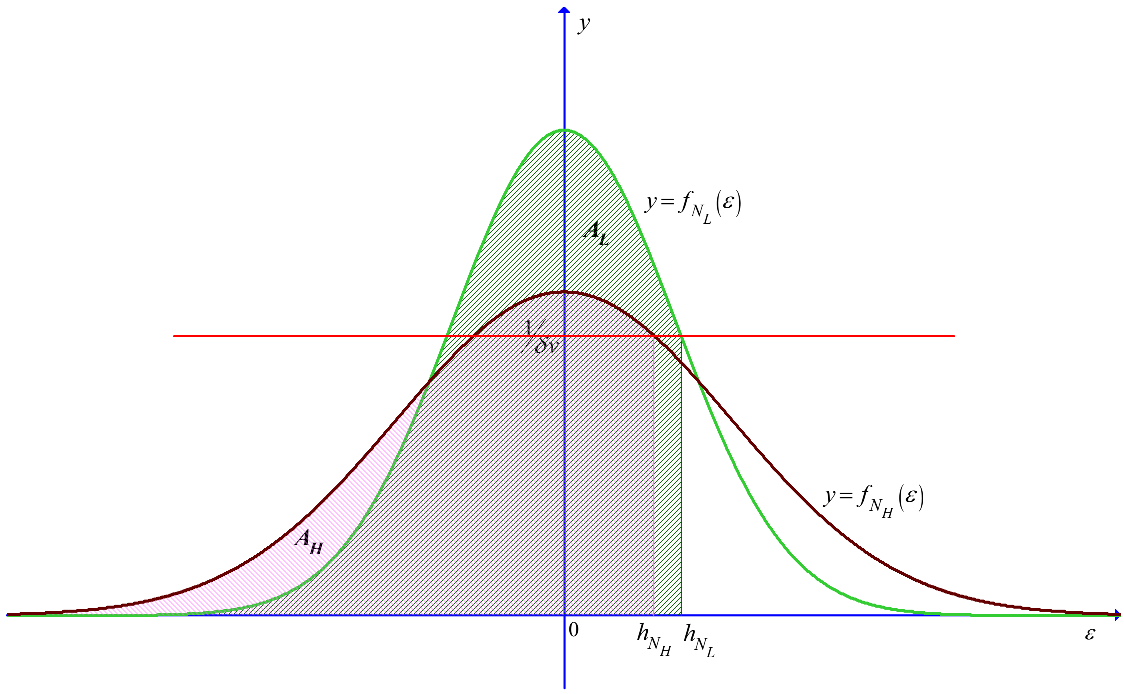

Figure 4: Equilibrium Re–election Probabilities of the Politicians

In the proof, we show that the re–election probability of a politician is negatively associated with the population size of the province he controls. Therefore, we expect politicians to be replaced more frequently in the more highly populated provinces.

For instance, assume that there are two provinces with population sizes N_L and N_H, where $N_L < N_H$. Figure 4 illustrates the distribution of the noise ε in these provinces. The curve with the greater variance belongs to the noise in the highly populated province. Proposition 9 implies that in equilibrium, the politician who controls the province with population size N_L is re–elected with probability $F_{N_L}(h_{N_L})$, while the politician who controls the province with population N_H is re–elected with probability $F_{N_H}(h_{N_H})$. By definition of h_N (equation 34), it is the positive ε that solves $f_N(\varepsilon) = \frac{1}{\delta v}$. Accordingly, h_{N_L} and h_{N_H} are labeled in the figure. $F_{N_L}(h_{N_L})$ is equal to the area A_L, and $F_{N_H}(h_{N_H})$ is equal to the area A_H. That is, the equilibrium re–election probabilities of the politicians in the provinces with low and high population sizes are equal to areas A_L and A_H, respectively. In proposition 11, we prove that $A_L > A_H$. Therefore, the politician who controls the province with low population size has a greater re–election probability than the politician who controls the province with a high population size. Consequently, we expect the politicians to be replaced more frequently in the highly populated provinces.

Based on the argument in the previous section, as political accountability increases, the politicians have greater re–election probabilities and they are replaced less frequently. Therefore, proposition 11 implies that political accountability is lower in more highly populated cities.

2.4 Empirical Analysis

Predictions of our model yield the empirical implication that the re–election probability of a politician is negatively related to the size of the province that he controls. In this section, we test this negative implication.

We work with the data of the Turkish municipal elections covering the 1963 – 2004 period. During this period, some of the counties became provinces for several reasons, including population growth and change in sociological conditions. This in turn resulted in an increase in the number of provinces in Turkey. Currently, there are 81 provinces in Turkey. The municipalities that we choose are the city centers of these 81 provinces. There are nine consecutive elections in this period.

We formed an indicator function, α, to trace whether the politicians were re–elected. In every province, we score the result of each election from the set $\{0, 1\}$. If the politician is re–elected, the indicator function is assigned a score of 1, otherwise it gets a score of 0. In province i, let w_t denote the identity of the elected politician at election time t. Our indicator function α can be represented as follows.

$$\alpha_{i,t} = \begin{cases} 1 \text{ if } w_t = w_{t-1} \\ 0 \text{ if } w_t \neq w_{t-1} \end{cases}$$

In nine elections, the maximum possible number for being re–elected is eight. In our data set the maximum number for being re–elected is five. The minimum number for being re–elected is zero and the average of it is 2.44.[4]

Our approach to the problem is to regress the indicator function on size of provinces and province specific dummies which is equivalent to estimating panel data fixed effects regression. The specification is given as following.

$$\alpha_{i,t} = c + \beta N_{i,t} + u_i + \epsilon_{i,t}$$

In this regression, c is constant, β represents the responsiveness of provincial size, u_i is the province specific effects, and $\epsilon_{i,t}$ are the error terms. $N_{i,t}$ is the population size of the province i at election time t.[5] We employ positive monotonic log–transformation to the variable N. The results of the regression are given in Table 1.

[4] Data for the municipal election results of the provinces are gathered from the reports of the High Election Council (YSK) and the website *www.yerelnet.org.tr,* which is operated within the context of the YEREP Project of the State Planning Organization (DPT).

[5] Data for the population sizes of the provinces are collected from the State Institute of Statistics (DIE) and the website *www.yerelnet.org.tr.*

Table 1: Estimations

β	-0.306 (0.073*)
constant	4.292 (0.955*)

Figures in the parenthesis are robust standard errors.

* indicates significance at the 99% confidence level.

Table 1 confirms the predictions of the model: the re–election probability of a politician is negatively related to the size of the province that he controls. An increase in the size of the province by one percent implies a reduction in the re–election probability of the politician by 0.31 percent.

2.5 Conclusion

This chapter analyzes the relation between the size of provinces (electorate) and the political accountability by using the re–election probabilities of politicians. The central idea of this chapter can be stated as follows. The electorate re–elects the politician and allows him to stay in office if and only if the politician is accountable. In more highly populated provinces, the signal of the electorate about the policies of the politician is noisier. Consequently, in these provinces, since the politicians are less able to justify their policies, they are less accountable to the electorate and voted out of the offices more frequently.

We presented a model on the public control of the politician in which the electorate's signal about the politician's decisions is noisier in more highly populated provinces. The model implies that the re–election probability of a politician is decreasing in the size of the province he controls. We test the prediction of the model by using a panel of nine municipal elections for 81 provinces in Turkey for the period 1963–2004. After controlling province specific effects, we observe that the re–election probability of the politician is decreasing in the size of the province. An increase in the size of the province by one percent implies a reduction in the re–election probability of the politician by 0.31 percent.

Based on the predictions of the model and the empirical support in favor of these predictions, we state that the re–election probability of a politician is decreasing in size of the province. Consequently, we expect politicians to be less accountable and replaced more frequently in more highly populated provinces.

2.6 Appendix

Proof of Proposition 8

The politician's best response to the voter's strategy with threshold $\underline{\theta}$ is to choose the policy that maximizes his expected payoff. The politician's expected payoff $U_N(\lambda|\underline{\theta})$ is given in (33), with the first and second derivatives of it being as follows.

$$\begin{aligned} U_N'(\lambda|\underline{\theta}) &= N\tau\left[-1+\delta v f_N(\underline{\theta}-\lambda)\right] \\ U_N''(\lambda|\underline{\theta}) &= N\tau\left[-\delta v f_N'(\underline{\theta}-\lambda)\right] \end{aligned} \tag{35}$$

From (35), if $f_N(\underline{\theta}-\lambda) < \frac{1}{\delta v}$, then the expected payoff of the politician is strictly decreasing in λ, and so the politician's best response is to select $\lambda = 0$ for any $\underline{\theta}$.

$$f_N(\varepsilon) = \frac{1}{\sqrt{2\pi N}\sigma} e^{-\frac{1}{2N}\left(\frac{\varepsilon}{\sigma}\right)^2} \tag{36}$$

$f_N(\varepsilon)$ is explicitly written above. It has the maximum value at $\varepsilon = 0$. Hence, if $f_N(0) < \frac{1}{\delta v}$, then the best response of the politician is to choose $\lambda = 0$ for any $\underline{\theta}$. Since the politician's strategy $\lambda = 0$ constitutes the best response to any of the voter strategies, in equilibrium the voter will choose any $\underline{\theta} \in \mathbb{R}$ and the politician selects $\lambda = 0$.

Therefore, the bound $\underline{v}$ is equal to $\frac{1}{\delta f_N(0)}$. By using (36), $\underline{v}$ can be written as follows.

$$\underline{v} = \frac{\sqrt{2\pi N}\sigma}{\delta}$$

Assume that $v \geq \underline{v}$. By solving $U_N'(\lambda|\underline{\theta}) = 0$ for λ, we see that $U_N(\lambda|\underline{\theta})$ has two extremum points. The first extremum point is at $\lambda = \underline{\theta} - h_N$ and the second one is at $\lambda = \underline{\theta} + h_N$.[6]

[6] h_N is defined in (34) as follows.

$$h_N = g_N^{-1}\left(\frac{1}{\delta v}\right) \text{ where } g_N(\varepsilon) = \begin{cases} f_N(\varepsilon) & \text{if } \varepsilon \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Note that if $\varepsilon < 0$, then $f_N(\varepsilon)$ is strictly increasing and $f_N'(\varepsilon)$ is positive. If $\varepsilon > 0$, then $f_N(\varepsilon)$ is strictly decreasing and $f_N'(\varepsilon)$ is negative. Accordingly,

$$U_N''(\underline{\theta} - h_N|\underline{\theta}) > 0 \text{ and } U_N''(\underline{\theta} + h_N|\underline{\theta}) < 0.$$

It is seen that $U_N(\lambda|\underline{\theta})$ has a local minimum point at $\lambda = \underline{\theta} - h_N$ and a local maximum point at $\lambda = \underline{\theta} + h_N$. Also, if $\lambda \in [\underline{\theta} - h_N, \underline{\theta} + h_N]$, then $U_N'(\lambda|\underline{\theta})$ is positive, so $U_N(\lambda|\underline{\theta})$ is strictly increasing. If $\lambda < \underline{\theta} - h_N$ or $\lambda > \underline{\theta} + h_N$, then $U_N'(\lambda|\underline{\theta})$ is negative, so $U_N(\lambda|\underline{\theta})$ is strictly decreasing.

The voter sets the threshold $\underline{\theta}$ to induce the politician to select a higher λ. Since $U_N(\lambda|\underline{\theta})$ has a local maximum at $\lambda = \underline{\theta} + h_N$, if the voter sets threshold $\underline{\theta}$ as $1 - h_N$, then the politician's best response is to choose λ as follows.

$$\lambda = \begin{cases} 0 & \text{if} \quad U_N(0|1-h_N) > U_N(1|1-h_N) \\ 1 & \text{otherwise} \end{cases} \tag{37}$$

By using (33), $U_N(0|1-h_N)$ and $U_N(1|1-h_N)$ can be written as follows.

$$\begin{aligned} U_N(0|1-h_N) &= N\tau[1 + \delta v - \delta v F_N(1-h_N)] & (38) \\ U_N(1|1-h_N) &= N\tau[\delta v - \delta v F_N(-h_N)] & (39) \end{aligned}$$

Recall that $h_N = g_N^{-1}\left(\frac{1}{\delta v}\right)$, so h_N also depends on the parameter v. We define $D_N(\cdot)$ as $D_N(v) = U_N(0|1-h_N(v)) - U_N(1|1-h_N(v))$. Then, by using (38) and (39), it can be written as follows.

$$D_N(v) = N\tau(1 - \delta v[F_N(1-h_N(v)) - F_N(-h_N(v))]) \tag{40}$$

The politician's best response when the voter chooses $\underline{\theta}$ as $1 - h_N$ can be rewritten from (37) as follows.

$$\lambda = \begin{cases} 0 & \text{if} \quad D_N(v) > 0 \\ 1 & \text{if} \quad D_N(v) \leq 0 \end{cases}$$

The first derivative of $D_N(v)$ is as follows.

$$\begin{aligned} D_N'(v) &= -\delta N\tau\left[F_N(1-h_N(v)) - F_N(-h_N(v))\right] \\ &\quad +\delta N\tau v h_N'(v)\left[f_N(1-h_N(v)) - f_N(-h_N(v))\right] \end{aligned}$$

Note that $h_N'(v) > 0$ and $[F_N(1-h_N(v)) - F_N(-h_N(v))] > 0$. Also, if $h_N(v) < \frac{1}{2}$, then $[f_N(1-h_N(v)) - f_N(-h_N(v))] < 0$. Consequently, if $h_N(v) < \frac{1}{2}$, then $D_N(v)$ is strictly decreasing.

Let $h_N(v_1)$ be equal to $\frac{1}{2}$. Then, v_1 can be written as follows.

$$v_1 = \frac{\sqrt{2\pi N}\sigma}{\delta} e^{-\frac{1}{8N\sigma^2}} \ ,$$

Note that if $v < v_1$, then $h_N(v) < \frac{1}{2}$. Therefore, if $v < v_1$, then $D_N(v)$ is strictly decreasing.

Also if $v > v_1$, then $[F_N(1-h_N(v)) - F_N(-h_N(v))] > \frac{1}{\delta v}$, thus $D_N(v) < 0$. Note that $h_N(\underline{v}) = 0$ and $[F_N(1) - F_N(0)] < \frac{1}{\delta \underline{v}}$, thus $D_N(\underline{v}) > 0$.

In summary, then, $D_N(v)$ is continuous, strictly decreasing in the interval $[\underline{v}, v_1]$ with boundary values $D_N(\underline{v}) > 0$ and $D_N(v_1) < 0$. Therefore, there exists a $\overline{v} \in [\underline{v}, v_1]$ such that $D_N(\overline{v}) = 0$. i.e.,

$$D_N(\overline{v}) = N\tau\left(1 - \delta\overline{v}\left[F_N(1-h_N(\overline{v})) - F_N(-h_N(\overline{v}))\right]\right) = 0.$$

If $v > \overline{v}$, then $D_N(\overline{v}) < 0$. So if the voter sets $\underline{\theta}$ as $1 - h_N$, then the politician's best response is to choose $\lambda = 1$. Therefore, if $v > \overline{v}$, the voter is able to induce the politician to choose $\lambda = 1$, hence in any SPE $\lambda = 1$. $\square$

Proof of Proposition 9

Note that we restrict our attention to $v \in (\underline{v}, \overline{v})$. Suppose that the voter chooses a strategy with threshold $\underline{\theta}$. In the proof of proposition 8, we described the politician's expected payoff $U_N(\lambda|\underline{\theta})$: It is strictly decreasing if $\lambda < \underline{\theta} - h_N$ and has a local minimum at $\lambda = \underline{\theta} - h_N$. It is strictly increasing in the interval $\lambda \in [\underline{\theta} - h_N, \underline{\theta} + h_N]$ and has a local maximum at $\lambda = \underline{\theta} + h_N$. If $\lambda > \underline{\theta} + h_N$, then $U_N(\lambda|\underline{\theta})$ is strictly decreasing. Also, we showed that if $\underline{\theta} \geq 1 - h_N$, then the politician's best response, given that $v < \overline{v}$, is to choose $\lambda = 0$.

Therefore, if $\underline{\theta} \leq -h_N$ or $\underline{\theta} \geq 1 - h_N$, the politician's best response is to select $\lambda = 0$. Thus, choosing a threshold $\underline{\theta} \leq -h_N$ or $\underline{\theta} \geq 1 - h_N$ cannot be an optimal strategy of the voter if there is a possibility of inducing the politician to choose a positive λ. If $\underline{\theta} \in [-h_N, 1 - h_N]$, then the politician's best response is as follows.

$$\lambda = \begin{cases} 0 & \text{if} \quad U_N(0|\underline{\theta}) > U_N(\underline{\theta} + h_N|\underline{\theta}) \\ \underline{\theta} + h_N & \text{otherwise} \end{cases}$$

We define $\chi(\cdot)$ as follows.

$$\chi(\underline{\theta}) = U_N(0|\underline{\theta}) - U_N(\underline{\theta} + h_N|\underline{\theta})$$

The best response of the politician in the case of $\underline{\theta} \in [-h_N, 1 - h_N]$ can be rewritten as follows.

$$\lambda = \begin{cases} 0 & \text{if} \quad \chi(\underline{\theta}) > 0 \\ \underline{\theta} + h_N & \text{if} \quad \chi(\underline{\theta}) \leq 0 \end{cases}$$

The voter wants the politician to choose higher λ, thus, the voter's best response to the politician's best response is to choose a $\theta^* \in [-h_N, 1 - h_N]$, which solves

$$\max_{\{\underline{\theta}\}} \underline{\theta} + h_N \quad \text{subject to: } \chi(\underline{\theta}) \leq 0. \tag{41}$$

Recall from the proof of Proposition 8, the condition that $v < \overline{v}$ implies that $h_N < \frac{1}{2}$. Since $U_N(\lambda|\underline{\theta})$ is strictly increasing in the interval $[\underline{\theta} - h_N, \underline{\theta} + h_N]$, if $\underline{\theta} = h_N$, then $U_N(0|h_N)$ is smaller than $U_N(2h_N|h_N)$. Thus, $\chi(h_N) < 0$.

By using the expected payoff function of the politician (equation 33), $\chi(\underline{\theta})$ can be written as follows.

$$\chi(\underline{\theta}) = N\tau\left[\underline{\theta} + h_N - \delta v\left(F_N(\underline{\theta}) - F_N(-h_N)\right)\right]$$

The derivative of $\chi(\underline{\theta})$ is as follows.

$$\chi'(\underline{\theta}) = N\tau\left[1 - \delta v f_N(\underline{\theta})\right] \tag{42}$$

Notice that by definition of h_N (equation 34), if $\underline{\theta} > h_N$, then $f_N(\underline{\theta}) < \frac{1}{\delta v}$. Thus, (42) implies that if $\underline{\theta} > h_N$, then $\chi(\underline{\theta})$ is strictly increasing.

In summary, then, $\chi(h_N) < 0$, $\chi(1 - h_N) > 0$ and $\chi(\cdot)$ is strictly increasing in the interval $[h_N, 1 - h_N]$. Therefore, there exists a unique $\theta^* \in [h_N, 1 - h_N]$ such that

$$\chi(\theta^*) = 0,$$

which also solves the maximization problem (41). Thus, in the unique SPE, the voter chooses threshold θ^* and the politician chooses policy $\lambda = \theta^* + h_N$.

In equilibrium, the re–election probability of the politician is equal to the following.

$$\begin{aligned} \alpha_N(\theta^* + h_N|\theta^*) &= 1 - F_N(\theta^* - (\theta^* + h_N)) \\ &= 1 - F_N(-h_N) \end{aligned}$$

By using the symmetry of the probability distribution function of the normal distribution, the re–election probability of the politician can be rewritten as follows.

$$\alpha_N\left(\theta^* + h_N|\theta^*\right) = F_N\left(h_N\right)$$

□

Proof of Proposition 10

In the proof of Proposition 9, we showed that if the voter chooses a strategy with a threshold greater than θ^*, then the politician's best response is to choose $\lambda = 0$. If the voter chooses a strategy $\underline{\theta} \in [-h_N, \theta^*]$, then the politician's best response is $\lambda = \underline{\theta} + h_N$.

If the politician chooses a strategy $\lambda \in [0,1]$, then any strategy of the voter is weakly optimal because the politician and his opponent are identical in terms of their ability and preferences.

Therefore, for $\underline{\theta} \in [-h_N, \theta^*]$, the following strategies are the best responses to each other and constitute an equilibrium: The voter chooses strategy with threshold $\underline{\theta}$ and the politician chooses policy $\lambda = \underline{\theta} + h_N$. Thus, $\lambda \in [0, \underline{\theta} + h_N]$ can be supported as the politician's policy in equilibria. □

Proof of Proposition 11

Proposition 9 implies that the equilibrium reelection probability of a politician who controls a province with population size N equals to $F_N\left(h_N\right)$, where

$$F_N\left(h_N\right) = \frac{1}{\sqrt{2\pi N}\sigma}\int\limits_{-\infty}^{h_N} e^{-\frac{1}{2N}\left(\frac{\varepsilon}{\sigma}\right)^2} d\varepsilon. \tag{43}$$

By changing the variable ε with$\sqrt{N}x$ in (43), we obtain

$$F_N\left(h_N\right)=\frac{1}{\sqrt{2\pi}\sigma}\int\limits_{-\infty}^{\frac{h_N}{\sqrt{N}}}e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}dx. \tag{44}$$

(44) implies that $F_N\left(h_N\right)=F(z)$ where $F\left(\cdot\right)$ is the cumulative distribution function of the normal distribution, with mean zero, variance σ^2, and $z=\frac{h_N}{\sqrt{N}}$. Also, by definition of h_N,

$$f_N\left(h_N\right)=\frac{1}{\delta v}=\frac{1}{\sqrt{2\pi N}\sigma}e^{-\frac{1}{2\sigma^2}\left(\frac{h_N}{\sqrt{N}}\right)^2}. \tag{45}$$

From (45), it can be written as follows.

$$\frac{1}{\delta v}=\frac{1}{\sqrt{2\pi N}\sigma}e^{-\frac{1}{2\sigma^2}z^2} \tag{46}$$

It is seen from (46) that z is inversely related to N. As N increases, z decreases and consequently, $F(z)$ decreases. Hence, the re–election probability of the politician is decreasing in N. $\square$

Chapter 3: Two–Sided Matching with Restrictions on Stating Preferences

3.1 Introduction

In this chapter, we examine the effects of the introduction of restrictions on the statement of preferences in a two–sided matching model with incomplete information. The model is similar to the process adopted for college admissions in Turkey.

In Turkey, the process of college admissions is centralized. There is a student placement office that assigns the students to the departments of the colleges. Every year this office administers an examination to every student who wishes to enroll in a college. Each student receives a score and a rank according to his performance on this examination. The higher the performance of a student in the examination, the higher is his rank. After receiving the ranks, each student completes a preference form and submits it to the student placement office. The departments have unanimous preferences – students with higher ranking are always preferred. Finally, the students are assigned to the departments by a pre–announced mechanism by using the students' preference forms and preferences of the departments.

Students may have different preferences over the departments. Each student only knows his preferences and has a prior belief about others' preferences. In this process, we are focusing on the point that the student placement office does not allow the students to order his preferences freely. There is a fixed number and the students can submit ordering, at most, this number of departments in the preference form. Since a student cannot declare his preferences for some departments that he may want to enroll, he must choose the departments in the preference form strategically.

In Turkey each year, approximately 1.5 million students take the national examination offered by the student placement office and there are more than 4,000 departments accepting the students. The student placement office restricts the students to submitting preferences a maximum of 18 departments. That means the students are exposed to a significant restriction when they are stating their preferences for departments.

If there is no restriction on the statement of preferences, this college admission problem has a unique *stable* matching. In this matching, the higher–ranked student is assigned to his top choice department, the second highest student is assigned to his top choice department from the available ones, and so on. We show that the restrictions on stating preferences, together with incomplete information of the students about the others' preferences, can result in unstable matching between the departments and the students.

Section 3.2 describes the model, section 3.3 presents the results, and section 3.4 concludes.

3.2 Model

There are n students and m departments. $S = \{s_1, ..., s_n\}$ denotes the set of students, $D = \{d_1, ..., d_m\}$ denotes the set of departments, and $C = \{c_{d_1}, ..., c_{d_m}\}$ denotes the set of capacities of the departments where c_{dj} is the capacity of d_j.

The students take a test and according to their performance on the test, each student receives a rank. Let s_i denote the student with rank i. Let $P(d_j)$ be the preference of the d_j over students. The departments have unanimous preferences over the students. They prefer a higher–ranked student over a lower ranked one. Formally, the preference of d_j is $P(d_j) = \langle s_1, s_2, ..., s_n \rangle$ for $j = 1, ..., m$.

Let $u(s_i)$ be the utility of s_i over the departments. We assume that for any s_i, $u(s_i)$ is drawn from a distribution as follows: $u(s_i) = u_k$ with probability $f(u_k)$ for $k = 1, .., K$ where $u_k = (u_{k_1}, ..., u_{k_m})$. A student with utility vector u_k gets u_{k_j} utility if he is placed to d_j.

The nature draws the utilities of the students over departments. Any student observes his utility over departments and his rank, but cannot observe the utilities of the other students. The preferences of the departments and the probability distribution of the students' utilities are common knowledge.

Each student submits a preference form in which he can state the ordering of, at most, A departments. Let o_i denote the ordering of s_i (in his preference form) and o_{-i} denote the orderings of other students. $o_i \in O$, where O is the set of possible orderings. Note that there are $\frac{m!}{(m-A)!}$ possible orderings.

There is a matching mechanism G that takes the orderings of the students and creates a matching μ between the students and departments. At matching μ, each student is either assigned to a department or assigned to himself. If a student is assigned to himself, then he is not accepted by any of the departments.

$G(o_1, o_2, ..., o_n)$ creates the matching μ by using the following algorithm. Starting with the highest–ranked student, in each iteration, one student is assigned to the first available department in his stated preference form. If all the departments in the student's preference form are unavailable, i.e., they were already assigned to other students, then the student will be assigned to himself. The algorithm continues with the next highest ranked student and concludes after the iteration for the lowest–ranked one.

Let $X_i : \{u(s_i) \rightarrow O\}$ be the strategy set of s_i that has $\|O\|^K = \left[\frac{m!}{(m-A)!}\right]^K$ strategies. Let F be the cumulative distribution function of the density f.

We define the matching game with incomplete information as follows.

$$\Gamma = \left(S \cup D, C, \{X_i\}_{i \in \{1,..,n\}}, \{u(s_i)\}_{i \in \{1,..,n\}}, \{P(d_j)\}_{j \in \{1,..,m\}}, F, G\right)$$

Example 1 *Assume there are five students* $(n = 5)$, *three departments* $(m = 3)$, *and the capacity of each department is one. The number of allowed preferences is two* $(A = 2)$ *and a student's utility over departments has three possibilities* $(K = 3)$ *as follows.*

$$u(s_i) = \begin{cases} u_1 = (3, 2, 1), & f(u_1) = 0.6 \\ u_2 = (2, 3, 1), & f(u_2) = 0.3 \\ u_3 = (2, 1, 3), & f(u_3) = 0.1 \end{cases} \quad for\ \forall s_i$$

For example, if a student's utility is u_1, *he gets three utility if he is assigned to department 1, two utility if he is assigned to department 2, and one utility if he is assigned to department 3. If a student is not assigned to any of the departments, he gets zero utility.*

Preference of a department over students is

$$P(d_j) = \langle s_1, s_2, s_3, s_4, s_5 \rangle \quad for\ \forall d_j.$$

There are $\frac{3!}{(3-2)!} = 6$ *possible orderings.*

$$O = \{\{1, 2\}, \{1, 3\}, \{2, 1\}, \{2, 3\}, \{3, 1\}, \{3, 2\}\}$$

Hence, any student has $6^3 = 216$ *strategies.*

Let $\mu(s_i)$ denote the department to which s_i is assigned under matching μ.

Definition 2 *Matching* μ *is* stable *if there is no student–department pair* $\{s_i, d_j\}_{s_i \in S, d_j \in D}$, *such that* s_i *prefers the* d_j *to* $\mu(s_i)$, *and* d_j *prefers the* s_i *to at least one of the students who is assigned to it under the matching* μ*. If such a pair exists, then we say that it* blocks *the matching* μ*.*

3.3 Results

Gale–Shapley (1962) showed that there always exists at least one stable matching in a two–sided matching problem by introducing an algorithm (the deferred acceptance procedure).

Proposition 12 *There exists a unique stable matching in the college admission problem described above. In this matching, the highest–ranked student is assigned to his top choice, the second highest one is assigned to his top choice among the available departments, and so on.*

The uniqueness is a consequence of the unanimous preferences of the departments. Hereafter μ^* denotes the unique stable matching.

Proof. The existence of a stable matching is due to Gale–Shapley(1962). Assume that there is another stable matching μ. Then, $\mu(s_1) = \mu^*(s_1)$. If this is not true, s_1 and $\mu^*(s_1)$ blocks μ. Also, $\mu(s_2) = \mu^*(s_2)$. Otherwise, s_2 and $\mu^*(s_2)$ blocks μ. By continuing so, $\mu(s_n) = \mu^*(s_n)$. Therefore, $\mu = \mu^*$. $\square$

Proposition 13 *If there is complete information (*$u(s_i)$ *is common knowledge among students), then* G *generates the matching* μ^**, even if* $A = 1$*.*

Proof. If $u(s_i)$ is common knowledge, then any student knows the others' preferences. Assume $A = 1$. The first–ranked student will choose his top choice and be assigned to that department $(\mu^*(s_1))$. The student with rank two infers the first–ranked student's placement, and his best strategy is to choose his top choice from remaining departments, which is $\mu^*(s_2)$. If he chooses another strategy, either he will not be assigned to any of the departments or he prefers $\mu^*(s_2)$ to the department to which he will be assigned under that strategy. In general, the student with rank k, $(k \leq n)$, knows the preferences and best strategies of other students, so he infers the first $(k-1)$–ranked students' placements. Consequently, his best strategy is to choose his top choice from available departments, which is $\mu^*(s_k)$. If he chooses another strategy, either he will choose an unavailable department and be assigned to himself, or he will be assigned to a department that he prefers less than $\mu^*(s_k)$. □

Proposition 14 *If there is no restrictions to stating preferences (i.e., $A = m$), then G generates the unique stable matching μ^*.*

Proof. If a student states his true preferences, he will be assigned to his most preferred department among the available ones when the algorithm turn comes to him. So Proposition 13 implies that G generates the matching μ^*. Hence, if we show that all students state their true preferences, then we prove the proposition.

Assume o_i be the true preference ordering of s_i and $G(o_i, o_{-i}) = \mu$ for $\exists\ o_{-i}$. Suppose that $G\left(o_i^{'}, o_{-i}\right) = \mu'$ for $\exists\ o_i^{'}$ and $\mu \neq \mu'$. We must show that s_i prefers $\mu(s_i)$ to $\mu'(s_i)$. Suppose that s_i prefers $\mu'(s_i)$ to $\mu(s_i)$. Since $G\left(o_i^{'}, o_{-i}\right) = \mu'$, $\mu'(s_i)$ is available when the algorithm turn comes to s_i. Hence, under his true preferences, s_i can not be assigned to $\mu(s_i)$, contradicting $G(o_i, o_{-i}) = \mu$. □

Definition 3 *Equilibrium for the game* Γ *with incomplete information is*

$$\left\{ \{x_i\}_{i\in\{1,..,n\}}, \{\beta_i\}_{i\in\{1,..,n\}}, \mu^e \right\},$$

where $\{x_i\}$ *is the set of the strategies of the students,* $\{\beta_i\}$ *is the set of beliefs of the students about higher–ranked students' ordering choices, and the matching outcome* μ^e *which is generated by algorithm G under strategies* $\{x_i\}_{i\in\{1,..,n\}}$ *such that :*

1) $x_i \in X_i$ *maximizes the expected utility of* s_i *under the the belief* β_i*, for* $\forall s_i \in S$*, and*

2) the beliefs of the students must be derived from strategies according to the Bayes' rule.

Proposition 15 *If there is a restriction on the statement of preferences* $(A < d)$, *the equilibrium outcome of the game* Γ *with incomplete information can be unstable.*

Proof. We present here an example in which $\mu^e \neq \mu^*$ and another one where $\mu^e = \mu^*$. In the example 1, equilibrium strategies, beliefs and matching outcome are the following. (We assumed that students select the low–indexed ordering when they are indifferent among the orderings.)

Equilibrium strategies

$$\begin{aligned}
x_1 &= \{1,2\} \text{ if } k=1, \ \{2,1\} \text{ if } k=2, \ \{3,1\} \text{ if } k=3 \\
x_2 &= \{1,2\} \text{ if } k=1, \ \{2,1\} \text{ if } k=2, \ \{3,1\} \text{ if } k=3 \\
x_3 &= \{2,3\} \text{ if } k=1, \ \{2,3\} \text{ if } k=2, \ \{1,3\} \text{ if } k=3 \\
x_4 &= \{1,2\} \text{ if } k=1, \ \{2,1\} \text{ if } k=2, \ \{1,2\} \text{ if } k=3 \\
x_5 &= \text{any strategy}
\end{aligned}$$

Equilibrium beliefs

$$\begin{aligned}
\beta_2 &= (0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0) \\
\beta_3 &= \begin{pmatrix} 0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0 \\ 0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0 \end{pmatrix} \\
\beta_4 &= \begin{pmatrix} 0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0 \\ 0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0 \\ 0.0\ 0.1\ 0.0\ 0.9\ 0.0\ 0.0 \end{pmatrix} \\
\beta_5 &= \begin{pmatrix} 0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0 \\ 0.6\ 0.0\ 0.3\ 0.0\ 0.1\ 0.0 \\ 0.0\ 0.1\ 0.0\ 0.9\ 0.0\ 0.0 \\ 0.7\ 0.0\ 0.3\ 0.0\ 0.0\ 0.0 \end{pmatrix}
\end{aligned}$$

The element in row a and column b of the matrix β_i reveals the belief of the s_i about the probability that student a chooses the ordering o_b where

$$o_1 = \{1,2\}, o_2 = \{1,3\}, o_3 = \{2,1\}, o_4 = \{2,3\}, o_5 = \{3,1\}, o_6 = \{3,2\}.$$

For example, student 4 believes that student 3 choose the ordering $\{1,3\}$ with probability 0.1, and the ordering $\{2,3\}$ with probability 0.9 and the others with probability zero.

Equilibrium outcome

Suppose that nature draws the utilities of the students as follows.

$$\begin{aligned}
u(s_1) &= u_2 = (2,3,1) \\
u(s_2) &= u_3 = (2,1,3) \\
u(s_3) &= u_1 = (3,2,1) \\
u(s_4) &= u_1 = (3,2,1) \\
u(s_5) &= u_2 = (2,3,1)
\end{aligned}$$

The unique stable matching μ^* is the following.

$$\mu^* = \left\{ \begin{pmatrix} s_1 \\ d_2 \end{pmatrix}, \begin{pmatrix} s_2 \\ d_3 \end{pmatrix}, \begin{pmatrix} s_3 \\ d_1 \end{pmatrix}, \begin{pmatrix} s_4 \\ \varnothing \end{pmatrix}, \begin{pmatrix} s_5 \\ \varnothing \end{pmatrix} \right\}$$

The equilibrium outcome μ^e is the following.

$$\mu^e = \left\{ \begin{pmatrix} s_1 \\ d_2 \end{pmatrix}, \begin{pmatrix} s_2 \\ d_3 \end{pmatrix}, \begin{pmatrix} s_3 \\ \varnothing \end{pmatrix}, \begin{pmatrix} s_4 \\ d_1 \end{pmatrix}, \begin{pmatrix} s_5 \\ \varnothing \end{pmatrix} \right\}$$

In matching μ^*, s_3 is assigned to d_1 and s_4 is assigned to himself. On the other hand, in matching μ^e, s_4 is assigned to d_1 and s_3 is assigned to himself. s_3 prefers d_1 instead of being assigned to himself and d_1 prefers s_3 to s_4. Therefore, $\{s_3, d_1\}$ blocks the equilibrium matching outcome. μ^e is not a stable matching.

Now, suppose that nature draws the utilities of the students as follows.

$$\begin{aligned} u(s_1) &= u_1 = (3,2,1) \\ u(s_2) &= u_2 = (2,3,1) \\ u(s_3) &= u_1 = (3,2,1) \\ u(s_4) &= u_3 = (2,1,3) \\ u(s_5) &= u_2 = (2,3,1) \end{aligned}$$

Then, the unique stable matching is equal to the equilibrium matching outcome.

$$\mu^* = \mu^e = \left\{ \begin{pmatrix} s_1 \\ d_1 \end{pmatrix}, \begin{pmatrix} s_2 \\ d_2 \end{pmatrix}, \begin{pmatrix} s_3 \\ d_3 \end{pmatrix}, \begin{pmatrix} s_4 \\ \varnothing \end{pmatrix}, \begin{pmatrix} s_5 \\ \varnothing \end{pmatrix} \right\}$$

$\square$

3.4 Conclusion

This chapter analyzed the effects of the introduction of restrictions on the statement of preferences in a two–sided matching model with incomplete information. The model is similar to the process used for college admissions in Turkey. In Turkey, the college admissions is centralized and a student placement office assigns the students to the departments of the colleges according to the preferences of the departments and the preference forms submitted by the students. Departments have unanimous preferences – students with higher ranking in the national examination are always preferred. Students are exposed to a restriction on the statement of the preferences; each student can state a preference ordering over a limited number of departments in the preference form. We demonstrated that the restriction on statement of the preferences can result in unstable matching between the departments and the students.

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