Commun.Fac.Sci.Univ.Ank.Series A1
Volume 59, Number 2, Pages 25–36 (2010)
ISSN 1303–5991

KINEMATICAL MODELS OF THE LOCK MOTIONS

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Abstract. In this study, mathematical modelling of lock and key mechanisms is focused and the kinds of motions of the structures are studied. The basic process principle of the lock and key mechanisms mathematically modelled in mechanic and kinematic are brought up. In this modelling, it is shown mutually as the movement the moving part produces or the movement the part setting into motion (K, A).

1. INTRODUCTION

1.1. **Rotation.** If $X = (x, y)$ are the coordinates of a point $P \in \mathbb{R}^2$ in the moving body measured in the coordinate frame M, then the coordinates of P measured in coordinate frame F can be given by

$$D : F \to M.$$

This transformation is given by

$$\vec{X} = [A]\vec{x} + d,$$

where $\vec{x}$ is the coordinate vector of a point in M and $\vec{X}$ is the coordinate vector of the same point but measured in F. If the dimension of the moving body is n (usually $n = 2$ or 3), then $[A]$ is an $n \times n$ matrix and d is n-dimensional vector. Let $n = 2$, so

$$[A] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, d = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}.$$

The pair (A, d) defines this transformation, and is called a planar displacement [2]. The matrix $[A]$ has $[A]^T$ as its inverse, therefore it is an orthogonal matrix; and, because its determinant is 1, it defines a rotation. It is interesting to examine the 2×2 orthogonal matrices that are not rotations [2].

Received by the editors Oct. 14, 2009, Accepted: May. 25, 2010.
2000 *Mathematics Subject Classification.* 70B15.
Key words and phrases. Lock, key, motion, kinematic, rotation, transition.

Let $R_\theta(0)$ be the θ degree rotation leaving the O point fixed. Then the rotation equations for n=2 can be given by

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

In brief, it can be written as

$$\vec{X} = R_\theta \vec{x}$$

where R_θ matrix is a matrix having the $R_\theta^T = R_\theta^{-1}$ property and R_θ is an orthogonal matrix.

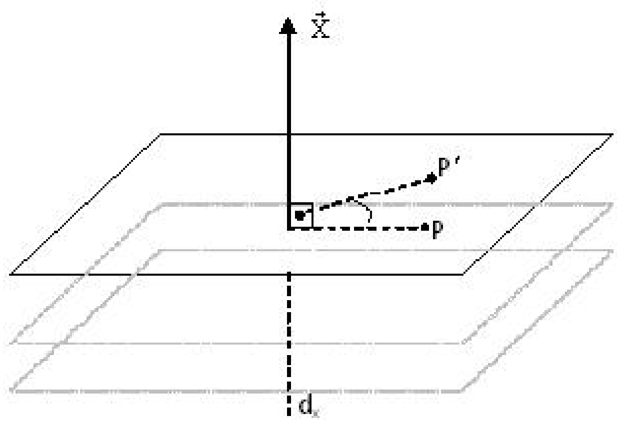

Figure 1. The plane evident with X_2, X_3.

The d_x line evident with $\vec{X}$ vector is called the rotation axis of the rotation R.

Definition 1.1. Let $f : I \subset \mathbb{R} \to \mathbb{R}$ show a given function. The matrix A determined by

$$[A] = \begin{bmatrix} \cos((-1)^{[|f(x)|]}.\theta(x)) & \sin((-1)^{[|f(x)|]}.\theta(x)) \\ \sin((-1)^{[|f(x)|]}.\theta(x)) & \cos((-1)^{[|f(x)|]}.\theta(x)) \end{bmatrix}$$

is called code matrix.

Example 1.2. Any rotation series for five handled can be shown by identified code matrix as: If $f : \mathbb{R} \to \mathbb{R}$, let

$$f(x) = \begin{cases} 1, & x = 0 \\ 2, & x = \frac{1}{2} \\ 3, & x = 1 \\ 4, & x = 2 \\ 1, & x = \frac{1}{4} \end{cases}$$

and then,

$$[A] = \begin{bmatrix} \cos((-1)^{([|f(x)|]}.2\pi x) & \sin((-1)^{[|f(x)|]}.2\pi x) \\ \sin((-1)^{[|f(x)|]}.2\pi x) & \cos((-1)^{[|f(x)|]}.2\pi x) \end{bmatrix}$$

$$[A_1] = \begin{bmatrix} \cos 0^0 & \sin 0^0 \\ -\sin 0^0 & \cos 0^0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_2] = \begin{bmatrix} \cos \pi & \sin \pi \\ -\sin \pi & \cos \pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[A_3] = \begin{bmatrix} \cos(-2\pi) & \sin(-2\pi) \\ -\sin(-2\pi) & \cos(-2\pi) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_4] = \begin{bmatrix} \cos(4\pi) & \sin(4\pi) \\ -\sin(4\pi) & \cos(4\pi) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_5] = \begin{bmatrix} \cos(-\pi/2) & \sin(-\pi/2) \\ -\sin(-\pi/2) & \cos(-\pi/2) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

is obtained.

2. THE MATHEMATICAL MODELLING OF THE LOCK MOTIONS

In this chapter, first, the lock mechanisms will be categorized by being studied in terms of motion kinds. There are basically two motions fullfilling the locking process. These are the motions that the moving part does and that put into motion does.Accordingly, the lock and key duet makes up a mechaism by moving bound to each aother. The mutual motion of this duet is kinematically based on the rotational and transitional motions. As the combination of rotations and translations, it is seen that different kinds of motions appear. Accordingly, if the key and lock duet (K, A) is expressed, it is understood that

$$(K, A) : (H_1, H_2)$$

H_1 : The motion that the tumbler in the lock does

H_2: The motion that the mechanism ensuring the tumbler to move

Here H_1 and H_2 are combination of one or more of the motions of T: transition, R: rotation, RT: rotational transition, DH: degenerate motion, V: screw motion. According to all those, the lock mechanisms can be grouped like the following in point of kinematic view with 6 different motion kinds:

1. $(K, A) : (T, T)$ Lock motion,
2. $(K, A) : (R, R)$ Lock motion,
3. $(K, A) : (T, RT)$ Lock motion,
4. $(K, A) : (R, V)$ Lock motion,
5. $(K, A) = (DH, T)$ Lock motion,
6. $((K_1, K_2, K_3, K_4), A) : ((T_1, T_2, T_3, T_4), R)$ Lock motion.
7. Multi and total motion of the lock mechan

3. THE STUDY OF THE KINEMATICAL MODELS OF THE LOCK MOTIONS

3.1. $(K, A) : (T, T)$ **Lock motion.** The lock is the one having the simplest structure in the lock mechanism motion kinds. In this kind of motion, both the part taking over the locking function and the motion applied on this part are directional motion. When whole of the mechanism is regarded, it is observed that the motion appeared is a transitional motion. The model of the concerning mechanism is,

$$(K, A) : (T, T).$$

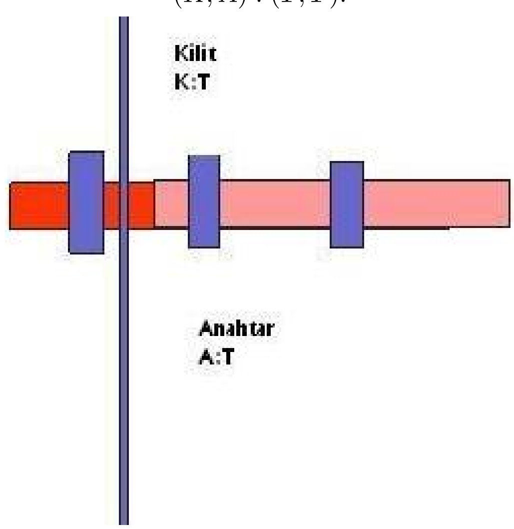

Figure 2. Bolted lock mechanism.

3.2. $(\mathbf{K}, \mathbf{A}) : (\mathbf{R}, \mathbf{R})$ **Lock Motion.** In this mechanism, the motion starts by means of a key. The structure putting the tumbler into motion creates a rotation motion. Accordingly, the motion that the tumbler brings up is a rotation. The model of this system can be given with

$$(K, A) : (R, R).$$

The matrix form of the motion with center $0 = (0, 0)$ is given by

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

The motion belonging to this system is as in Figure 3

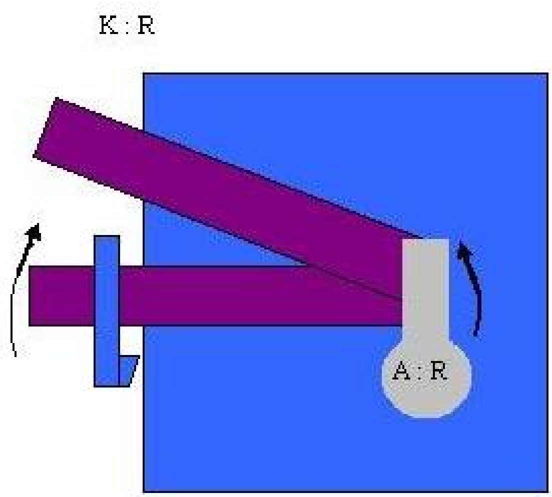

Figure 3. Rotational motion lock mechanism

3.3. $(\mathbf{K}, \mathbf{A}) : (\mathbf{T}, \mathbf{RT})$ **Lock Motion.** In this kind of motion, there is a different kinematic fixing. The mechanism does a whole transition motion but the source of this transition motion is the rotational motion that the key creates. Since the motion part of the moving part of the lock is limited, the key actually does a transitional rotation.

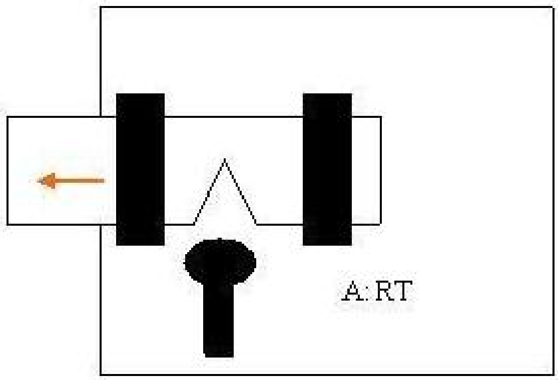

Figure 4 Rotational transition motion system

Let the rotation axis of the key be as z axis, the rotation point be as the origin point and the rotation plane as $x \circ y$ plane. The closing point of the lock goes from the location A to location B by the effect of the lock. Since the joint will move in a certain slide on the lock, it creates a transition motion. The transition vector is $(L, 0)$. Each point on the key joint has the rotation of

$$(x, y) \to (x - L, y).$$

Accordingly, the matrix of this motion is as

$$[K] = \begin{bmatrix} \cos\theta & \sin\theta & d_1 \\ -\sin\theta & \cos\theta & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} x - L\sin\theta \\ y_0 \\ 1 \end{bmatrix}.$$

Since the θ degree is evident, (d_1, d_2) transition vector must be calculated,

$$\begin{aligned} x\cos\theta + y_0\sin\theta + d_1 &= x - L\sin\theta \\ -x\sin\theta + y_0\cos\theta + d_2 &= y_0 \end{aligned}$$

And then, d_1 and d_2 transitions are obtained as,

$$\begin{aligned} d_1 &= (1-\cos\theta)x - (y_0 + L)\sin\theta \\ d_2 &= xs \in \theta + (1-\cos\theta)y_0 \end{aligned}$$

Accordingly, the model is evident as

$$(K, A) : (T, RT).$$

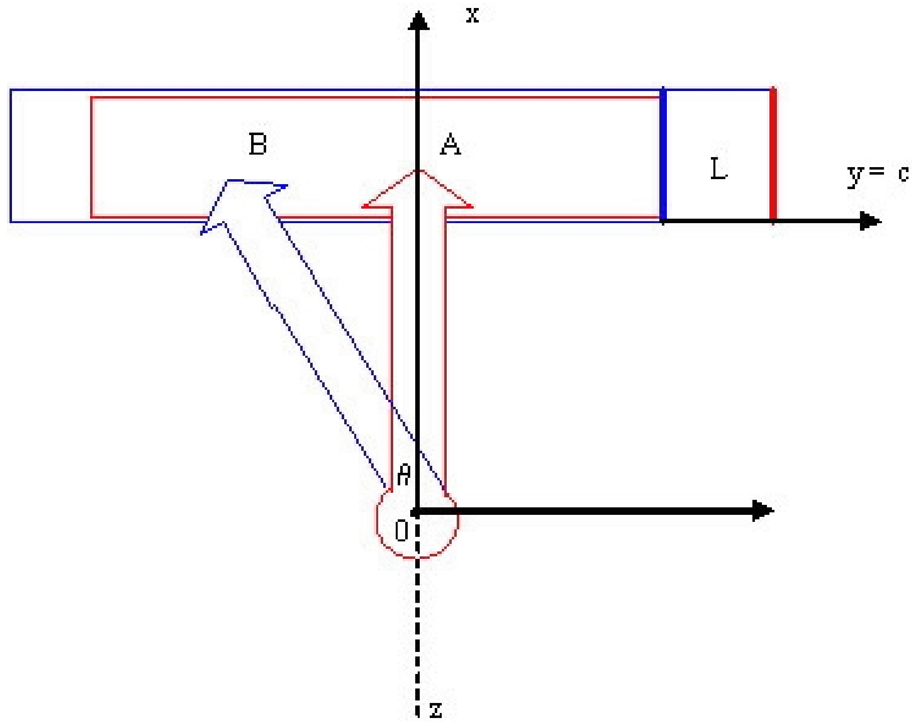

Figure 5. Rotational transition motion system -2

3.4. $(\mathbf{K}, \mathbf{A}) : (\mathbf{R}, \mathbf{V})$ **Lock Motion.** Another kind of the lock motions is seen in the screw motioned lock mechanism.Key is in a shape of screw here, and does a helix motion. The key entering the system with a helix motion results in releasing squeezing the string mechanism. Such a motion allows the system to be opened. Therefore,

$$(K, A) : (R, V)$$

equality can be written for this motion.

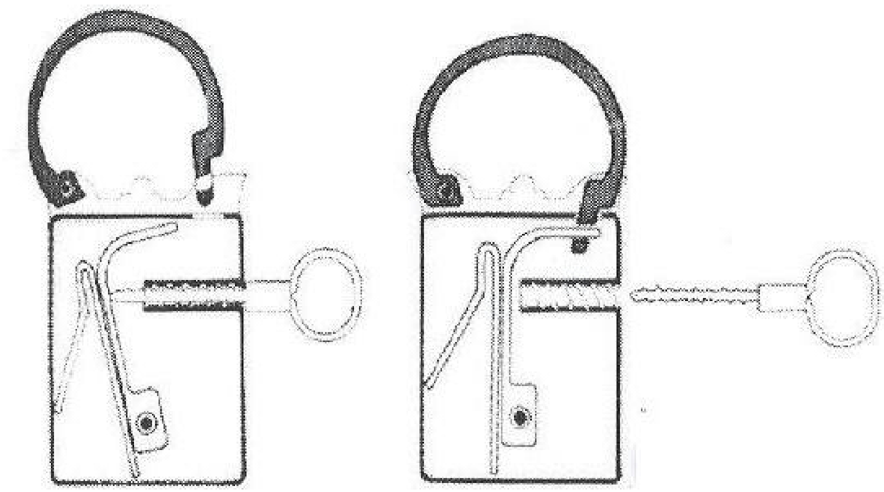

Figure 6. Inside structure of the screw motioned lock mechanism [5]

3.5. $(\mathbf{K}, \mathbf{A}) = (\mathbf{DH}, \mathbf{T})$ **Lock Motion.** The piece having the role of a key in this mechanism leaves the pieces hindering the mechanism to be opened inactive by forcing on the free positioned the string. Therefore, the pieces in the mechanism can be separated from each other and the system gets opened. A different kind of motion which we can also call obstacle canceller, kind of lock shows itself in this lock system.

3.6. $((\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3, \mathbf{K}_4), \mathbf{A})$ **:** $((\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3, \mathbf{T}_4), \mathbf{R})$ **Lock Motion.** It is possible to name this kind of motion as multi motioned-single motioned mechanism. A multi joined lock mechanism with a motion from a single center is simultaneously a mechanism in which a four-directed lock motion is done. Here four systems are put into motion in a single stage with a single lock motion shown in type 3. This system has not only sprang mechanisms but also pinned and obstacled key kinds developed today. The spring in the system allows the mechanism to be opened even without using a lock pick. However, pinned and obstacled keys cancel this problem

Rotational transition motion in four different direction is taken as K_1, K_2, K_3, K_4. In this case we have

$$[K_1]_= \quad \begin{bmatrix} \cos\theta & \sin\theta & d_1 \\ -\sin\theta & \cos\theta & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} x - L\sin\theta \\ y_0 \\ 1 \end{bmatrix}$$

$$\begin{aligned} x\cos\theta + y_0, \sin\theta + d_1 &= x - L\sin\theta \\ -x\sin\theta + y_0\cos\theta + d_2 &= y_0 \end{aligned}$$

Therefore, d_1 and d_2 transitions are

$$\begin{aligned} d_1 &= (1-\cos\theta)x - (y_0+L)\sin\theta \\ d_2 &= x\sin\theta + (1-\cos\theta)y_0 \end{aligned}$$

The second rotation + transition motion
From the equations

$$[K_2] = \begin{bmatrix} \cos\theta & \sin\theta & d_1 \\ -\sin\theta & \cos\theta & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ -y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} -x+L\sin\theta \\ -y_0 \\ 1 \end{bmatrix}$$

we have

$$\begin{aligned} x\cos\theta - y_0, \sin\theta + d_1 &= -x+L\sin\theta \\ -x\sin\theta - y_0\cos\theta + d_2 &= -y_0 \end{aligned}$$

d_3 and d transitions are found as

$$\begin{aligned} d_3 &= (-1-\cos\theta)x + (y_0+L)\sin\theta \\ d_4 &= x\sin\theta + (-1+\cos\theta)y_0 \end{aligned}$$

The third rotation + transition motion
From the equations

$$[K_3] = \begin{bmatrix} \cos\theta & \sin\theta & d_1 \\ -\sin\theta & \cos\theta & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x_0 \\ y-L\sin\theta \\ 1 \end{bmatrix}$$

we have

$$\begin{aligned} x_0\cos\theta + y\sin\theta + d_1 &= x_0 \\ -x_0\sin\theta + y\cos\theta + d_2 &= y - -L\sin\theta \end{aligned}$$

Thus d_5 and d_6 transitions are found as

$$\begin{aligned} d_5 &= (1-\cos\theta)x_0 - -y\sin\theta \\ d_6 &= (1-\cos\theta)y + (x_0-L)\sin\theta \end{aligned}$$

The fourth rotation + transition motion
From the equations

$$[K_4] = \begin{bmatrix} \cos\theta & \sin\theta & d_1 \\ -\sin\theta & \cos\theta & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -x_0 \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -x_0 \\ -y+L\sin\theta \\ 1 \end{bmatrix}$$

we have

$$\begin{aligned} -x_0\cos\theta + y\sin\theta + d_1 &= -x_0 \\ x_0\sin\theta + y\cos\theta + d_2 &= -y+L\sin\theta \end{aligned}$$

d_7 and d_8 transitions are as the following

$$\begin{aligned} d_7 &= (-1+\cos\theta)x_0 - -y\sin\theta \\ d_8 &= (-1-\cos\theta)y + (L-x_0)\sin\theta \end{aligned}$$

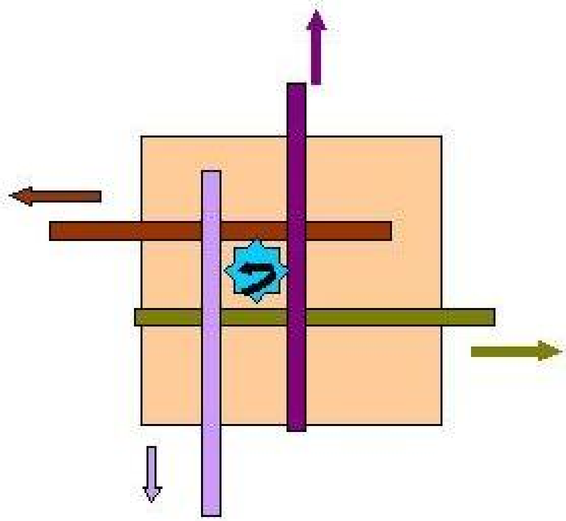

Figure 7. Rotational transition motion in four different direction from a single center

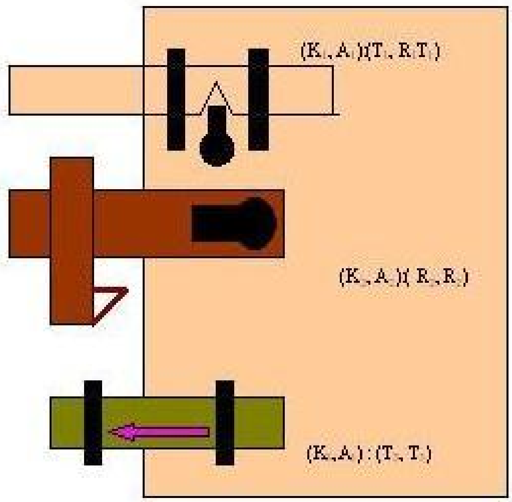

Figure 8. Summary of three different motion systems.

4. CONCLUSION

In this study, kinds of motions in different kinds of lock mechanisms have been studied by making a kinematical modelling description The relations and differences between the kinds of motions were brought in. The matrices and functions belonging to rotations and transitions in the systems were described. $[K_1]$, $[K_2]$, $[K_3]$, $[K_4]$ matrices belonging to the four different rotational transitions and $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ transitions from a single center have been obtained.

ÖZET: Bu çalışmada kilit ve anahtarların matematik ve kinematik modellemeleri çalışıldı. Matematik ve kinematik açıdan modellemenin temel işlemi öncelikle mekanizmaları sınıflandırmaktır. Modellemede kilit ve anahtarlar (K, A) gösterimi kullanılarak sınıflandırıldı.

References

[1] Bottema, O., Roth, B., 1979. *Theoretical kinematics.* North-Holland Publishing Company, New York. 557.

[2] McCarthy, J. Micheal, 1990. *An Introduction to Theoretical Kinematics.* MIT Press, Cambridge. 130.

[3] Niku, Saed B.Niku, 2001. *Introduction to Robotics. Prentice Hall Press,* New Jersey. 349.

[4] O'Neil, Peter V.,1987.*Advanced Engineering Mathematics.* Wadsworth Publishing Company,California. 535.

[5] Tanavoli, P., 1976. *Locks from Iran, Smithsonian Institution, Iran. 151.*

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