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# The Sequence Space $l(p,s)$ And Related Matrix Transformations

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SUMMARY

In this paper, our main purpose is to define and to investigate the sequence space $l(p, s)$ and to determine the matrices of classes $(l(p, s), l_\infty)$ and $(l(p, s), c)$ where $l_\infty$ and c are respectively the spaces of bounded and convergent complex sequences and for $p = (p_k)$ with $p_k > 0$, the space $l(p, s)$ is defined by

$$l(p, s) = \{ x = (x_k) : \sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k} < \infty , s \geq 0 \}.$$

1. Let $A = (a_{nk})$ be an infinite matrix of complex numbers $a_{nk}$ $(n, k = 1, 2, \ldots)$ and v, w be two subsets of the space of complex sequences. We say that the matrix A defines a matrix transformations from v into w and denote it by writing $A \in (v, w)$, if for every sequence $x = (x_k) \in v$ the sequence $Ax = (A_n(x)) \in w$,

where $A_n(x) = \sum_{k=1}^{\infty} a_{nk} x_k$.

In this paper, our main purpose is to define and to investigate the sequence space $l(p, s)$ and to determine the matrices of classes $(l(p, s), l_\infty)$ and $(l(p, s), c)$, where $l_\infty$ and c are respectively the spaces of bounded and convergent complex sequences and for $p = (p_k)$ with $p_k > 0$, the space $l(p, s)$ is defined by

$$l(p, s) = \{ x = (x_k) : \sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k} < \infty, s \geq 0 \}.$$

Obviously, the sequence space

$$l(p) = \{ x = (x_k) : \sum_{k=1}^{\infty} |x_k|^{p_k} < \infty, p_k > 0 \}$$

which has been investigated by several authors **[1,3,5,7,]** is a special case of $l(p, s)$ which corresponds to $s = 0$. And $l(p, s) \supset l(p)$.

Throughout the paper the following well-known inequalities will be used frequently.

For any complex numbers a, b,

$$|a + b|^p \leq |a|^p + |b|^p \tag{1}$$

where $0 < p \leq 1$; and

$$|a \cdot b| \leq |a|^q + |b|^p \tag{2}$$

where $1 < p < \infty$ and $p^{-1} + q^{-1} = 1$. **N** will denote the set of natural numbers and **R** the set of real numbers.

Using the same kind of argument to that in **[4]**, we get that the necessary and sufficient condition for $l(p, s)$ to be linear is

$$0 < p_k \leq \sup_k p_k = H < \infty.$$

To begin with we can show that the space $l(p, s)$ is paranormed by

$$g(x) = \Big(\sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k}\Big)^{1/M}, \tag{3}$$

where $H = \sup_k p_k < \infty$, and $M = \max(1, H)$. Clearly, $g(\theta) = 0$ and $g(x) = g(-x)$, where $\theta = (0, 0, \ldots)$. Take any $x, y \in l(p, s)$. Since $p_k / M \leq 1$ and $M \geq 1$, using the Minkowski's inequality we have

$$\Big(\sum_{k=1}^{\infty} k^{-s} |x_k + y_k|^{p_k}\Big)^{1/M}$$

$$\leq \Big(\sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k}\Big)^{1/M} + \Big(\sum_{k=1}^{\infty} k^{-s} |y_k|^{p_k}\Big)^{1/M}$$

which shows that g is subadditive.

Finally, to check that the continuity of multiplication, let us take any complex $\lambda$. Then we have

$$g(\lambda x) = \Big(\sum_{k=1}^{\infty} k^{-s} |\lambda x_k|^{p_k}\Big)^{1/M} \leq \sup_k |\lambda|^{p_k/M} \cdot g(x).$$

Now, let $\lambda \to 0$ for any fixed x with $g(x) \neq 0$. Since $\sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k} < \infty$, there exists an integer $N > 0$, for $|\lambda| < 1$ and $\varepsilon > 0$, such that

$$\sum_{k=N+1}^{\infty} k^{-s} |\lambda x_k|^{p_k} < (\varepsilon/2)^M < \varepsilon/2. \tag{4}$$

Taking $|\lambda|$ sufficiently small such that $|\lambda|^{p_k} < \varepsilon/2 g(x)$ for $k = 1, 2, \ldots, N$; then we have

$$\sum_{k=1}^{N} k^{-s} |\lambda x_k|^{p_k} < \varepsilon/2. \tag{5}$$

(4) and (5) together implies that $g(\lambda x) \to 0$ as $\lambda \to 0$.

It is quite routine to show that $(l(p, s), d)$ is a metric space with the metric d defined by $d(x, y) = g(x - y)$ providing that $x, y \in l(p,s)$, where g is defined by (3). And using the similar method to that in [6] one can show that for $0 < m = \inf p_k \leq p_k \leq \sup_k p_k = H < \infty$, $l(p, s)$ is complete under the metric mentioned above.

We shall also say that $(e_k)$ is a Schauder base for $l(p, s)$, where $e_k$ is a sequence with 1 in the k th place and zero elsewhere.

2. Now we are going to give the following theorem by which the Köthe-Toeplitz dual of $l(p, s)$ will be determined.

THEOREM 1. (i). *If* $1 < p_k \leq \sup_k p_k = H < \infty$ *and* $p_k^{-1} + q_k^{-1} = 1$ *for* $k = 1, 2, \ldots$ *then*

$$l^{\dagger}(p,s) = \left\{ a = (a_k) : \sum_{k=1}^{\infty} k^{s(q_k-1)} N^{-q_k/p_k} |a_k|^{q_k} < \infty, \; s > 0, \text{ for some integer } N > 1 \right\}$$

(ii) *If* $0 < m = \inf_k p_k \leq p_k \leq 1$ *for each* $k = 1, 2, \ldots$ *then* $l^{\dagger}(p,s) = m(p,s)$, *where*

$$m(p,s) = \{ a = (a_k) : \sup_k k^s |a_k|^{p_k^{-1}} < \infty, \; s \geq 1 \}. \tag{6}$$

PROOF. (i). Let $1 < p_k \leq \sup_k p_k = H < \infty$ and $p_k^{-1} + q_k^{-1} = 1$ for each $k \in N$. Then take

$$E(p,s) = \left\{ a=(a_k) : \sum_{k=1}^{\infty} k^{s(q_k-1)} N^{-q_k/p_k} \quad |a_k|^{q_k} < \infty, \; s \geq 0, \; \text{for some integer } N > 1 \right\} \tag{7}$$

We now want to show that $l^{\dagger}(p, s) = E(p, s)$. Let $x \in l(p,s)$, $a \in E(p, s)$ and N be the associated number with a, Therefore, using the inequality (2), we get

$$|a_k \; x_k| \leq k^{s(q_k-1)} N^{-q_k/p_k} \; |a_k|^{q_k} + N \; k^{-s} \; |x_k|^{p_k}.$$

So $\Sigma \; |a_k \; x_k|$ is convergent which implies that $\Sigma \; a_k \; x_k$ converges, i. e., $a \in l^{\dagger}(p, s)$. In other words, $l^{\dagger}(p, s) \subset E(p, s)$.

Conversely, let us suppose that $\Sigma \; a_k \; x_k$ is convergent and $x \in l(p, s)$, but $a \notin E(p, s)$. Then we write that

$$\sum_{k=1}^{\infty} k^{s(q_k-1)} N^{-q_k/p_k} \quad |a_k|^{p_k} = \infty$$

for each $s \geq 0$ and for every $N > 1$. So we can find a sequence $0 = n(0) < n(1) < n(2) < \ldots$ such that for $\nu = 1, 2, \ldots$

$$M_\nu = \sum_{I(\nu)} k^{s(q_k-1)} (\nu+1)^{-q_k/p_k} \quad |a_k|^{q_k} > 1$$

where the sum $\sum_{I(\nu)}$ is taken over the range $n(\nu-1) + 1 \leq k \leq n(\nu)$.

Now, define a sequence $x = (x_k)$ as follows:

$$x_k = (\text{sgn } a_k) \; |a_k|^{q_k-1} \; k^{s(q_k-1)} \; (\nu+1)^{-q_k} \; M_\nu^{-1} \qquad ; k \in I(\nu)$$

$$x_k = 0 \qquad ; k \notin I(\nu).$$

Then we find that

$$\begin{aligned} \sum_{I(\nu)} a_k \; x_k &= \sum_{I(\nu)} |a_k|^{q_k} \; k^{s(q_k-1)} \; (\nu+1)^{-q_k} \quad M_\nu^{-1} \\ &= \sum_{I(\nu)} |a_k|^{q_k} \; k^{s(q_k-1)} \; (\nu+1)^{-q_k/p_k} \quad M_\nu^{-1} \; (\nu+1)^{-1} \\ &= (\nu+1)^{-1} \end{aligned}$$

but

$$\sum_{I(\nu)} k^{-s} |x_k|^{p_k} = \sum_{I(\nu)} k^{-s} |a_k|^{(q_k-1)p_k} k^{s(q_k-1)p_k} (\nu+1)^{-q_k \cdot p_k} M_\nu^{-p_k}$$

$$= \sum_{I(\nu)} |a_k|^{q_k} k^{sq_k} k^{-s} (\nu+1)^{-q_k/p_k} (\nu+1)^{-1-p_k} M_\nu^{-p_k}$$

$$\leq (\nu+1)^{-2} M_\nu^{-1} \sum_{I(\nu)} |a_k|^{q_k} k^{s(q_k-1)} (\nu+1)^{-q_k/p_k}$$

$$= (\nu+1)^{-2}$$

that is, $\Sigma\ a_k x_k$ diverges but $x \in l\ (p, s)$. And this contradicts to our assumption. So $a \in E\ (p, s)$, i.e., $l^\dagger\ (p, s) \subset E\ (p, s)$. Then combining these two results we get

$$l^\dagger\ (p,s) = E\ (p, s).$$

(ii). Let $0 < m = \inf_k p_k \leq p_k \leq 1$ for each $k \in \mathbf{N}$. Now we want to show that $l^\dagger\ (p, s) = m\ (p, s)$ where

$$m\ (p, s) = \{a = (a_k) : \sup_k k^s\ |a_k|^{p_k} < \infty,\ s \geq 0\}.$$

Suppose that $\Sigma\ a_k x_k$ converges and $x \in l\ (p, s)$ but $a \notin m\ (p, s)$. Then we can choose a sequence $1 \leq \nu\ (1) < \nu\ (2) < \ldots$ such that

$$(\nu\ (q))^s\ |a_{\nu(q)}|^{p_{\nu(q)}} \geq q^2 \quad (q = 1, 2, \ldots).$$

Then for a sequence $(x_k)$ defined by

$$x_k = a_k^{-1} \quad k = \nu\ (q), \qquad q = 1, 2, \ldots$$

$$x_k = 0 \quad k \neq \nu\ (q)$$

we get

$$\sum_{k=1}^{\infty} k^{-s}\ |x_k|^{p_k} = \sum_{q=1}^{\infty} (\nu(q))^{-s}\ |a_{\nu(q)}|^{-p_{\nu(q)}}$$

$$\leq \sum_{q=1}^{\infty} q^{-2} < \infty$$

but

$$\sum_{k=1}^{\infty} a_k\ x_k = \sum_{q=1}^{\infty} 1 = \infty$$

which is a contradiction. So $a \in m\ (p, s)$.

Conversely, let $a \in m(p, s)$ and $a \neq 0$. Let $\sup_k k^s |a_k|^{p_k} = B$, say. Then the series $\Sigma a_k x_k$ is convergent for $x \in l(p, s)$ providing that $\Sigma k^{-s} |x_k|^{p_k} \leq 1/B$. Because, the assumption $\sup_k k^s |a_k|^{p_k} = B$ gives the result $k^s |a_k|^{p_k} \leq B$ for each k. And considering the ineqvality $\Sigma k^{-s} x_k^{p_k} \leq 1/B$, we find that $k^{-s} |x_k|^{p_k} \leq 1/B$ for each k. Then multiplying these two results we obtain $|a_k x_k|^{p_k} \leq 1$ and $|a_k x_k| \leq |a_k x_k|^{p_k} \leq 1$, since $0 < p_k \leq 1$. Therefore $\Sigma a_k x_k$ converges, since

$$\sum_{k=1}^{\infty} |a_k x_k| \leq \sum_{k=1}^{\infty} |a_k x_k|^{p_k} \leq \sup_k k^s |a_k|^{p_k} \sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k} < \infty.$$

But, if $x \in l(p, s)$ then, since $l(p, s)$ is linear, we can find an integer $N > 1$ such that $\sum_{k=1}^{\infty} k^{-s} \left| \frac{x_k}{N} \right|^{p_k} \leq 1/B$. Therefore, the above discussion gives the convergence of $\Sigma a_k x_k / N$ and so $\Sigma a_k x_k$ is convergent, i.e., $a \in l^{\dagger}(p, s)$, which completes the proof of the theorem.

Let us now determine the continuous dual of $l(p, s)$ by the following theorem.

THEOREM 2. (i). *If* $1 < p_k \leq \sup_k p_k = H < \infty$ *for* $k = 1, 2, \ldots$ *then* $l^*(p, s)$. *i.e., the continuous dual of* $l(p, s)$, *is isomorphic to* $E(p, s)$ *which is defined by* (7).

(ii). *If* $0 < m = \inf_k p_k \leq p_k \leq 1$ *for each* $k = 1, 2, \ldots$ *then* $l^*(p, s)$ *is isomorphic to* $m(p, s)$ *which is defined by* (6).

PROOF. (i). Since $e_k$, $k = 1, 2, \ldots$ are the unit vectors of $l(p,s)$ then, for every $x$ in $l(p,s)$, we can write $x = \sum_{k=1}^{\infty} x_k e_k$, whence $f(x) = \sum_{k=1}^{\infty} a_k x_k$ for any $f$ in $l^*(p, s)$, where $f(e_k) = a_k$.

By Theorem 1 (i), the convergence of $\sum_{k=1}^{\infty} a_k x_k$ for every x in l (p, s) implies that $a \in E(p, s)$.

If $x \in l(p, s)$ and if we take $a \in E(p, s)$ then, by Theorem 1 (i), $\sum_{k=1}^{\infty} a_k x_k$ converges and clearly defines a linear functional on l (p,s). Using the same kind of argument to that in Theorem 1 (i) it is easy to check that

$$\left|\sum_{k=1}^{\infty} a_k x_k\right| \leq \left(\sum_{k=1}^{\infty} |a_k|^{q_k} N^{-q_k/p_k} k^{s(q_k-1)} + N\right) g(x)$$

whenever $g(x) \leq 1$, where $g(x) = \left(\sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k}\right)^{1/M}$ and

$p_k^{-1} + q_k^{-1} = 1$. Hence $\sum_{k=1}^{\infty} a_k x_k$ defines an element of $l^*(p, s)$. Obviously, the map $T : l^*(p, s) \to E(p, s)$ given by $T(f) = a$ is linear and bijective.

(ii) Since the sequence $(e_k)$ is a Schauder base for l (p, s), we can write $x = \sum_{k=1}^{\infty} x_k e_k$ for every $x \in l(p, s)$. Then, for every f in $l^*(p, s)$, $f(x) = \sum_{k=1}^{\infty} a_k x_k$, where $a_k = f(e_k)$. So, by Theorem 1 (ii), the convergence of $\sum_{k=1}^{\infty} a_k x_k$ for every $x \in l(p, s)$ implies that

$a \in m(p,s)$. Now, if $x \in l(p,s)$ and $a \in m(p,s)$ then $\sum_{k=1}^{\infty} a_k x_k$ converges by Theorem 1(ii) and, of course, defines a linear functional on l (p, s).

Now, we must show that $f(x) = \sum_{k=1}^{\infty} a_k x_k$ is continuous. Let $x \in l(p, s)$ and $\varepsilon > 0$ is given and $d(\theta, x) = g(x) \leq$

$\frac{\min(1, \varepsilon)}{B}$ where $B = \sup_k k^s \, |a_k|^{p_k} < \infty$. Then, by the same method used in Theorem 1 (ii), we see that $|f(x)| = |\sum_{k=1}^{\infty} a_k x_k| \leq \sum_{k=1}^{\infty} |a_k x_k| < \varepsilon$ which implies the continuity of f at the origin. So, f is continuous at every point of $l(p,s)$, since f is a linear functional on $l(p,s)$. Hence $\sum_{k=1}^{\infty} a_k x_k$ defines an element of $l^*(p,s)$. It is now evident that the map $T : l^*(p,s) \to m(p,s)$ given by $T(f) = a$ is a linear bijection.

3. In the following theorems we are going to characterized the matrix classes $(l(p,s), l_\infty)$ and $(l(p, s), c)$.

THEOREM 3. (i). *If* $1 < p_k \leq \sup_k p_k = H < \infty$ *for every* $k \in \mathbf{N}$ *then* $A \in (l(p, s), l_\infty)$ *if and only if there exists an integer* $D > 1$ *such that*

$$\sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} \; D^{-q_k} \; k^{s(q_k-1)} < \infty. \tag{8}$$

(ii) *If* $0 < m = \inf_k p_k \leq p_k \leq 1$ *for each* $k \in \mathbf{N}$, *then* $A \in (l(p, s), l_\infty)$ *if and only if*

$$K = \sup_{n,k} |a_{nk}|^{p_k} \; k^s < \infty. \tag{9}$$

PROOF. (i). **Sufficiency**. By using the inequality (2) we get

$$|a_{nk} x_k| \leq D \; [\, |a_{nk}|^{q_k} \; k^{s(q_k-1)} \; D^{-q_k} + |x_k|^{p_k} \; k^{-s} \,]$$

for every n. Then, if we take the sum in both sides over k from 1 to $\infty$ and consider the hypothesis, we obtain, for every n,

$$|\sum_{k=1}^{\infty} a_{nk} x_k| \leq \sum_{k=1}^{\infty} |a_{nk} x_k| < \infty,$$

i.e., $(A_n(x)) \in l_\infty$, whenever $x \in l(p, s)$.

**Necessity.** Suppose that $A \in (l(p, s), l_\infty)$ but that

$$\sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} N^{-q_k} k^{s(q_k-1)} = \infty$$

for every integer $N > 1$. Then $\sum_{k=1}^{\infty} a_{nk} x_k$ converges for every n and for every $x \in l(p, s)$, whence $(a_{nk})_{k=1,2}, \ldots \in l^{\dagger}(p, s)$ for every n. By Theorem 2 (i), it follows that each $A_n$ defined by $A_n(x) = \sum_{k=1}^{\infty} a_{nk} x_k$ is an element of $l^*(p, s)$. Since $l(p, s)$ is complete and since $\sup_n |A_n(x)| < \infty$ on $l(p, s)$, there exists by the uniform boundedness principle a number L independent of n and x, and a number $\delta < 1$ such that

$$|A_n(x)| \leq L \tag{10}$$

for every $x \in S[\theta, \delta]$ and every n, where by $S[\theta, \delta]$ we denote the closed sphere in $l(p, s)$ with centre at the origin $\theta = (0, 0, \ldots)$ and radius $\delta$.

Now choose an integer $Q > 1$ such that

$$Q \delta^H > L.$$

By our assumption we have

$$\sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} = \infty$$

and so two cases are possible: either

$$\sum_{k=1}^{\infty} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} < \infty$$

for every $n \geq 1$ or there exists an $n \geq 1$ such that

$$\sum_{k=1}^{\infty} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} = \infty.$$

In the first case, there exists $n \geq 1$ such that

$$\sum_{k=1}^{\infty} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} > 2$$

and there exists $k_0 > 1$ such that

$$\sum_{k=k_o+1}^{\infty} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} < 1$$

whence

$$\sum_{k=1}^{k_o} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} > 1.$$

In the second case we may choose $k_o > 1$ such that

$$\sum_{k=1}^{k_o} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} > 1$$

so that in either case there exist an $n \geq 1$ and $k_o > 1$ such that

$$V = \sum_{k=1}^{k_o} |a_{nk}|^{q_k} Q^{-q_k} k^{s(q_k-1)} > 1. \tag{11}$$

We now define using (10) a sequence $x = (x_k)$ as follows:

$$x_k = \delta^{H/p_k} |a_{nk}|^{q_k-1} (\operatorname{sgn} a_{nk}) V^{-1} Q^{-q_k/p_k} k^{s(p_k-1)} \quad ; 1 \leq k \leq k_o$$

$$x_k = 0 \quad ; k > k_o$$

Then one can easily show that $g(x) \leq \delta$ but $|A_n(x)| > L$, which contradicts to (10). This completes the proof of Theorem 3 (i).

(ii) The sufficiency and the necessity can be proved respectively by the same kind of argument used in Theorem 2 (ii) and by the uniform boundedness principle.

THEOREM 4. (i). *Let* $1 < p_k \leq \sup_k p_k = H < \infty$ *for every* $k \in \mathbf{N}$. *Then* $A \in (l(p, s), c)$ *if and only if together with* (8) *the condition*

$$a_{nk} \to \alpha_k \qquad (n \to \infty,\ k \text{ fixed}) \tag{12}$$

*hold.*

(ii) *Let* $0 < m = \inf_k p_k \leq p_k \leq 1$ *for ever* $k \in \mathbf{N}$. *Then* $A \in (l(p,s), c)$ *if and only if the conditions (9) and (12) hold.*

PROOF. (i). The necessity of (12) can easily be obtain using the unit vector $e_k$. For the sufficiency we have, for every integer $r \geq 1$ and every n

$$\sum_{k=1}^{r} |a_{nk}|^{q_k} D^{-q_k} k^{s(q_k-1)} \leq \sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} D^{-q_k} k^{s(q_k-1)} < \infty.$$

So,

$$\lim_{r\to\infty}\lim_{n\to\infty}\sum_{k=1}^{r} |a_{nk}|^{q_k} D^{-q_k} k^{s(q_k-1)} \leq \sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} D^{-q_k} k^{s(q_k-1)}$$

i.e.,

$$\sum_{k=1}^{\infty} |\alpha_k|^{q_k} D^{-q_k} k^{s(q_k-1)} < \sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} D^{-q_k} k^{s(q_k-1)} .$$

Hence $(\alpha_k) \in l^{\dagger}(p, s)$ and since also $(a_{nk})_{k=1,2,\ldots} \in l^{\dagger}(p, s)$ the series $\sum_{k=1}^{\infty} \alpha_k x_k$ and $\sum_{k=1}^{\infty} a_{nk} x_k$ converge for every n and for every $x \in l(p, s)$.

We can choose an integer $r \geq 1$ such that

$$\sum_{k=r+1}^{\infty} k^{-s} |x_k|^{p_k} < 1$$

whenever $x \in l(p, s)$. Then by the proof of Theorem 2 (i) and by the inequality (2) we have

$$\sum_{k=r+1}^{\infty} |a_{nk} - \alpha_k| \; |x_k|$$

$$\leq 2D\left[1+2\sup_n \sum_{k=1}^{\infty} |a_{nk}|^{q_k} D^{-q_k} k^{s(q_k-1)}\right]\left[\sum_{k=r+1}^{\infty} k^{-s} |x_k|^{p_k}\right]^{1/H}$$

which implies that

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} x_k = \sum_{k=1}^{\infty} \alpha_k x_k .$$

(ii) By the proof of Theorem 2 (ii) we get the proof of this part in a similar way to that in (i).

*REMARK.* To be able to get the necessary and sufficient condition for $A \in (l(p, s), c_0)$, where $c_0$ is the space of null sequences, it would be enough to take $\alpha_k = 0$ in the above theorem.

## ÖZET

Bu çalışmada amacımız, $p_k > 0$ olmak üzere $p = (p_k)$ dizisi için

$$l(p, s) = \left\{ x = (x_k) : \sum_{k=1}^{\infty} k^{-s} |x_k|^{p_k} < \infty , s \geq 0 \right\}.$$

ile tanımladığımız l (p, s) dizi uzayını sınırlı $p = (p_k)$ için incelemektir. Ayrıca $l_\infty$ ve c sırasıyla sınırlı ve yakınsak kompleks terimli dizilerin oluşturduğu dizi uzaylarını göstermek üzere $(l(p, s), l_\infty)$ ve $(l(p, s), c)$ matris sınıfları belirlenmiştir.

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